ICT and Mathematics: a guide to learning and teaching mathematics 11-19

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Context:
The guide has been designed to be of particular relevance and usefulness to groups such as the following:
- Those involved with organising programmes of continued professional development (CPD) for mathematics teachers in schools and colleges, particularly in connection with the NOF-funded ICT training and its follow-up.
- Those responsible for initial teacher training courses for new mathematics teachers.
- Teachers in schools and colleges receiving laptops under the current DfES Laptops for Teachers scheme, those who have received computers under previous schemes and those who have purchased their own equipment.
- LEA mathematics staff advising schools, colleges and teachers including KS3 strategy consultants, advisory teachers, advisers and inspectors.
- Those planning the deployment of ICT resources in school and college mathematics departments including senior managers, middle managers, heads of departments and mathematics teachers.
- Those schools and colleges with specialist status, such as TC Trust members, which includes targets for mathematics (technology, science, engineering, business & enterprise, mathematics & computing) and those making applications for such status.
- Those charged with developing future policy, materials and support involving ICT use in mathematics 11-19.

Structure:
The guide is written in a direct style addressing you, the reader. We, the authors, have tried to give you enough concrete information, illustrated by examples, to help you extend the use of ICT in supporting learning and teaching. We are conscious that we can only include a limited amount of information in such a guide, so we have tried to provide as many references to further useful sources of information as we can. Similarly we have tried to be as-up-to-date as we can with regard to the resources covered in the guide, but we are aware that this will change over time and that new technologies will be developed which may well impact on mathematics.

The guide is structured in three parts.
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Introduction

The government has made a huge investment in ICT in schools over the past few years and is committed to maintain it. One aspect of this investment has been the Continued Professional Development opportunities provided for all teachers in ICT through the training groups approved by the New Opportunities Fund (NOF). The rate at which secondary school subjects, especially mathematics, have taken up the use of ICT to support teaching and learning has not been as rapid as might have been expected. This could be for a number of reasons, and we hope that this guidance document might make a contribution to improving the situation.


It summarises the present situation and does not find the overall picture very bright!

2. The effect of government ICT initiatives on the quality of teaching and learning in mathematics varies considerably among schools. Overall, good practice remains uncommon.”

13. …although they know the general advantages of ICT use, many mathematics teachers remain unaware of the potential of specific software and tools: for example, the power of the data handling facilities on graphical calculators or the facility of graph plotting software to transform general shapes. They are less able to employ ICT to meet the different needs of pupils, and because of weak planning are more liable to be drawn in to teaching ICT skills to the detriment of the mathematics which the ICT is intended to support.

So don’t be downhearted if your department is not making as full use of ICT as it might – you are far from alone! Once you have read this document we trust that you will be in a much better position to plan a successful strategy for improving the situation for the benefit of your students and your mathematics staff alike.

Section 1: Planning the effective use of ICT in teaching and learning mathematics:

ICT is just another, albeit very powerful, resource which you, as a mathematics teacher, will need to consider when planning work for your students inside and outside mathematics lessons. Of course finance for resources is finite. But there has been a huge public investment in ICT hardware, software, support and training for education which is ongoing. Parents, students and teachers have a right to expect that teaching of all subjects in the curriculum should make the best possible use of ICT resources, and that means that the selection and deployment of ICT resources should be made to fit the needs of the subject – and not the reverse. The test of whether it makes sense to deploy ICT in mathematics is a simple one: “Does it benefit the students’ effective learning of mathematics?”

Developments in ICT have been very rapid, and in general costs have fallen considerably, so it is now the case that many homes contain more powerful ICT resources than are currently available to many teachers of mathematics in schools and colleges. Many common ICT tools have been developed for personal use, and their deployment in a teaching environment requires careful consideration. In business it is a common rule of thumb that hardware
accounts for less than half the investment in ICT, and that investment in hardware is of very little use without investment in appropriate software, as well as training and support.

Unfortunately a culture has arisen in education in which the provision of resources to support the acquisition of appropriate curriculum software lags far behind that of hardware, which often comes with a range of so-called “generic” software bundled-in. In section 2 we give a review of the many software tools which have already been developed to support teaching and learning mathematics. Most of these have been widely available for some considerable time at a relatively modest cost. Inspection evidence consistently shows very little use of appropriate ICT tools in mathematics teaching at all levels. Reasons for this may include: (i) lack of support and/or training, (ii) lack of funding devolved to mathematics departments, (iii) scepticism about benefits of using ICT and (iv) lack of knowledge of what is available. We hope this guidance will help enable you to articulate your needs in a way which may improve the situation with respect to (i) and (ii). If you feel you don't know what ICT resources are available, nor the benefits of their use to teachers and learners, then we hope that this guidance will contribute to improving your knowledge.

The TTA’s principal aim for the NOF-funded ICT training is to equip teachers with the necessary knowledge, skills and understanding to make sound decisions about when, when not, and how to use ICT effectively in teaching particular subjects. Our experience is that the effective use of ICT in mathematics teaching gives a powerful context which contributes to, and promotes, learning. The key to this is how you plan to integrate the use of appropriate ICT tools. When used well, such tools make your students the active participants in the learning. In the next section we will review a number of models of classroom use of ICT. In each case you need to be able to answer the question: “What does the ICT in this particular model of use bring to the learning?” The recent Ofsted report - “ICT in Schools” (2002) – contains the following useful statement in Paragraph 54:

Good teaching using ICT begins with clarity of purpose in its use. Most often this comes with experience, and through thoughtful planning and collaboration between teachers integrating ICT into a scheme of work. Once pupils are working on their own or in groups, effective teaching involves teacher intervention to make sure that the objectives of the lesson are met and that pupils do not become distracted by the technology.

Paragraph 90 of the same report discusses some of the options for ICT deployment in secondary school subject teaching:

The characteristics of good general secondary provision include:

- availability of different groupings of resources to match the needs of departments, for example computer rooms, clusters of machines and individual workstations around the site
- computers networked and well maintained with good Internet access from all workstations
- well-lit, comfortable computer rooms with sufficient space for pupils to work away from computers and for teachers to circulate and talk to individual pupils
- effective communication with the whole class using digital projectors or the capacity to control all the computers
- an efficient and equitable booking system for computer rooms.

Increasingly schools are exploring the possibility of greater flexibility in serving subject needs through the use of wireless links between banks of laptops, managed centrally and linked to the school network.
Of course different subjects will also have further styles of ICT use to consider – for example in mathematics we have hand-held devices such as graphical calculators and data-loggers, and whole-class teaching tools such as interactive whiteboards to consider.

Whatever style of hardware use we choose, the key issue will be the choice of software to support the learning and teaching of mathematics. Paragraph 41 of Ofsted’s earlier report, “ICT in Schools” (2001), advises that:

Less successful use of ICT in core subject teaching typically stems from weak links between the computer task and the lesson objectives. Too many teachers select software packages for their visual appeal rather than their relevance to lessons. In one case, a primary science lesson revolved around a science program, which, although quite stimulating, placed very few demands on pupils, who watched a simulation of materials dissolving, rather than carrying out experiments of their own.

The Ofsted ICT Secondary Mathematics report also has sections on managing ICT in mathematics and on staff development. Overall, the report appears to paint a rather depressing picture. The following extracts give a taste:

6. However, good, consistent and progressive use of ICT in mathematics is found in only a small minority of schools. In around two thirds of mathematics departments some use is made of ICT, but not to the extent that it is an established part of the curriculum.

16. The management of ICT developments within mathematics departments is generally weak. … most mathematics departments do not have ICT as a priority.

29. In the small minority of schools where ICT resources and accommodation are very good, these have a favourable effect on achievement in mathematics. However, even where mathematics departments have their own computer rooms, their use is not always well planned and in several cases they are insufficiently used.

While we have done our best to juggle our desire to give you as much helpful advice and information as possible within a document of tolerable length you will probably find it useful to have some other resources to hand when wanting to go rather further into aspects we can only touch on. Such resources include:

- recent editions of the ATM’s `Micromath’ journal (say from Vol 17 onwards)

1.1 Models of ICT deployment in schools and colleges

In recent years mathematics teaching has seen a renaissance of whole class teaching approaches. So clearly you need to know what sorts of ICT resources are available to support this. In many cases, though, you will also want the students themselves to have access to ICT resources. This might be through individual access, or working in pairs, or working in groups. Nearly all schools and colleges have special purpose ICT suites where students can have individual or paired access to computer resources. These may be available to subject departments through a booking system – but there are issues about such use which you will need to consider carefully. An alternative is to make ICT resources available within mathematics teaching rooms – and here, too, there are issues you need to think about. Of
course, learning is not restricted to timetabled lessons and need not take place solely in the school or college. So you need also to think about the access which students might have to resources away from lessons which can support their learning and your teaching.

We are fortunate that mathematics was one of the three subjects, along with Latin and Japanese, chosen to be the subject of a recent DfES funded trial project which has been extensively evaluated. Lessons from the mathematics project, based on 20 Year 7 classes and their teachers, should help inform us about each of the deployment issues above. In this project it was assumed that all the teachers and many of the pupils would have Internet access outside lessons. A central tenet of the project was that the classroom should provide for as flexible use of ICT as possible. In the photograph below you can see at the front, scanning from left to right, (a) an overhead projector on which there is a liquid crystal display panel for projecting a teacher’s model of a graphical calculator, (b) an interactive whiteboard mounted at a height suitable for pupil use and connected to the PC and printer at the front, as well as the ceiling-mounted data projector and (c) one of a further two PCs in the corner of the room. Not shown are (d) the teacher’s own laptop and printer, (e) the teacher’s graphical calculator, (f) the compatible data-logger (motion detector) and (g) the half-class set of a further 15 graphical calculators for pupil use.

Whole class work with ICT

The image may be generated by ICT hardware such as:
- a single PC or laptop
- hand-held technology such as a teacher’s graphical calculator
- another ICT device such as a digital camera or cam-corder.

It may be projected by display technology such as:
- a data-projector – either portable or permanently sited
- an overhead projector (OHP) and liquid crystal display (LCD) pad
- an interface to convert the input signal to a standard output such as PAL video.
It may be **displayed** on devices such as:

- a fabric screen
- an ordinary whiteboard suitable for use with marker pens
- an interactive whiteboard (IWB) designed for use with special styli and pens
- one or more large screens such as display monitors, TV sets, plasma displays…

Paragraph 69 of the 2001 Ofsted report refers to PCs for whole class displays:

*Many schools have invested in interactive whiteboards and computer projection equipment.* Generally these have led to more effective whole-class teaching, with all pupils able to see a large, clear display. Pupils too have made good use of this new equipment, for example in illustrating a prepared talk with the aid of presentational software. In particular, the use of interactive whiteboards is bringing benefits to pupils with SEN, for example by making teachers’ presentations clearer and more interesting, and by providing instant notes via printouts of the text and examples used in presentations. In an effort to provide more flexible access to ICT resources for pupils, some schools have increased the availability of **laptop computers**. Many teachers have benefited from the DfEE scheme for part-funding computers, although demand has outstripped the available funding. Other teachers have benefited significantly from Government schemes to provide laptops for personal use.

Paragraph 70 of the same report refers to hand-held devices such as graphical calculators:

*In specific subjects in secondary schools, NGfL funding has been used to purchase specialised ICT equipment.* … There is increasing availability of hand-held devices in **mathematics**, with more powerful calculators that can be used to display graphs, capture and analyse data, manipulate algebraic expressions and solve equations. These devices can significantly enhance mathematics teaching, especially where teachers can provide demonstrations via an overhead projector.

As we have seen, the DfES project made use both of PC displays projected via a ceiling-mounted data-projector onto an interactive whiteboard and graphical calculator (GC) displays projected via a conventional OHP onto any flat clear surface. If possible it is preferable to display either PC or GC displays onto a surface on which you can write, so that you can easily annotate diagrams etc. You can see further examples of the use of whole class displays with mathematics classes in each of the four case studies in the Secondary Mathematics part of disk 3 of the TTA’s ICT CD-ROM needs identification materials, obtainable from the Publications Line on 0845 606 0323. In Section 3 below we give more details about whole class displays such as interactive whiteboards.
ICT resources for whole class work can be used in a number of ways including mental oral work, demonstration, interactive whole class teaching, class discussion and plenary activities. If they are PC-based then, at a minimum, single user licences would be required to give access to dynamic geometry software (such as *Cabri* or *Geometer’s Sketchpad*), a programming language (such as *MSW Logo*), a graph plotter (such as *Omnigraph* or *Coypu*) and/or other powerful integrated mathematics software (such as *TI Interactive!*). This would allow teachers and pupils to make interactive use of ICT in a variety of ways, although pupils would only have limited opportunities for hands-on work. It could also be possible to access resources from the Internet (either ‘live’ or downloaded), such as games, simulations, data and animations. If the resources include hand-held devices then much of this software is already built in to graphical calculators. With so-called `flash ROM’ (such as in the TI-83 Plus) further applications can be downloaded and run, including spreadsheet, small software (such as *SMILE* programs), computer algebra and dynamic geometry.

**Individual, or paired, use of ICT**

In a minority of schools there is a mathematics room equipped with a suite of PCs on a network which supports a good base of curriculum software. In such a case the allocation of mathematics rooms can usually be adjusted so that each class has reasonable access to hands-on use at an appropriate time. More commonly, though, there will be one or more ICT suites in the school available for sharing between subject and ICT teaching using a booking system. In either case there are a number of issues concerning the layout of the room. Ideally it should provide:

- a whole-class display visible to all students
- scope for visual oral interaction between teacher and pupils in both one-to-one and whole class situations
- space for pupils to work away from the computers
- a range of software appropriate to the mathematics curriculum.

An alternative approach to giving all students hands-on access to ICT in their normal classroom is provided by having a set of portable battery-driven devices which could range, in descending order of cost, from a set of laptops (possibly linked wirelessly to a network), through palm-top computers (such as the Palm Pilot, Casio Cassiopeia and TI Voyage 200) to graphical calculators. There are advocates both of having enough for all pupils to use individually and for making pairs of students share. The model taken following the DfES project uses 20 TI-83 Plus graphical calculators – enough so that students can work in pairs, but also to support a few individual students who may, for example, be working at a different pace. The Y10 algebra case study on the TTA’s CD-ROMS illustrates a lesson including graphical calculators both for whole class work and for working in pairs. A major advantage of the use of portable ICT is that students can use it in their normal work-space, and make their own decisions about when, and when not, to use it.

Whatever means of access you have at your disposal you will still need to plan carefully for collaborative activities. Thought also needs to be given to practical issues such as:

- how students’ work may be saved for future access
- whether hard-copy devices should be available
- the possibility of students displaying their own work to the whole class.
Group work with ICT

It may be possible to have just a few PCs in the mathematics classroom which can be used for work with small groups of students – say 3 or 4 to a machine. If there are not enough for the whole class then some kind of ‘circus-style’ system of management will be required with some students working on tasks away from PCs while others work with them – maybe changing over during the lesson or for the next lesson. The kind of activity using the PC will require careful planning – ideally it should involve the students working collaboratively as a group to plan the next move involving the PC. For example this might be participating in an adventure game or simulation, or in a problem-solving or modelling task where the PC is just one of the available tools. While primary school teachers often adopt such circus-style of activity, with or without ICT, it is less common in secondary school mathematics and does involve careful planning.

Mixed models

Of course you may not be restricted to any one of the above models, in which case you will be able to make flexible decisions about the best way to deploy ICT to meet a particular curriculum objective. This was the resourcing model followed in the DfES project – which extends well beyond just Y7 to the whole 11-19 age-range.

In planning ICT use for a department you need to bear in mind how you can best provide an initial deployment of ICT resources which maintains flexibility and which allows you to continually improve the types of access as finances allow. For example if you already have access to a number of laptops for teachers’ use then you may want to aim to build up a stock of portable data-projectors so that they can be used for whole class work. Or you might want to plan a phased equipping of a number of mathematics classrooms with interactive whiteboards. In either case you might also want to build up a stock of graphical calculators which can be taken into any lesson. You need to be able to make a reasoned case for what you reckon is a realistic base from which to meet the ICT demands for organising the teaching and learning of your subject in the 21st Century. Remember that hardware is only
part of the story – you will need to plan also for acquiring appropriate software licences. Later in this section we give advice on management issues, and extensive advice on ICT products and software comes in Section 2. A useful summary of issues is given in Ronnie Goldstein’s article ‘Organise yourself’ in Micromath 17/2, page 48.

Summary

We hope that you have been able to gain an understanding of the main types of organisation of ICT resources for mathematics classes, and that you are aware of the range of issues associated with each. We conclude this section with some comments from the recent Ofsted reports on ICT in schools.

The following observations are extracted from paragraphs 91-93 of the Ofsted 2002 report:

*The suitability of accommodation varies considerably; a minority of schools have good levels of high-quality accommodation, equipment and software, for both general and subject-specific purposes. .. Many schools, however, find it difficult to complement these (ICT) lessons by sufficient use of ICT in other subjects.*

*Where they are unable to find space for a dedicated suite, schools often use mini-suites or clusters of computers in one space or distribute computers between classrooms. The management of the latter is difficult, though not impossible, requiring very organised teaching to ensure that each pupil has sufficient access to the computer over a period of time.*

*A minority of ... schools use laptops or hand-held computers to give flexibility of use in different classrooms. ... this approach also enables more flexibility in the way that ICT is used in the curriculum, for example by locating sufficient resources in the normal classroom to support work in ... subjects.*

Paragraph 42 of the 2001 Ofsted report observes that:

*Although the use of computer suites is growing, much of the teaching of IT takes place in ordinary classrooms. Individuals or groups of pupils are set ICT-based tasks, while most work away from computers. The increase in resources has meant that this practice has become easier to manage over a period of time. Teachers are becoming skilled in setting up such arrangements, but there remains a considerable variation in how well the pupils learn when working at computers. Successful classroom practices usually occur when teachers take care to ensure that pupils understand what they have to do and what they have to learn.*
1.2 Changes in teaching approaches

It is quite possible to use ICT in mathematics lessons in a way which does not throw into question your own teaching approaches. However the power of the medium is such that to restrict its use in this way is to deprive the students of access to a resource which could revolutionise their learning. It was to address this issue that the DfES’ mathematics curriculum IT support group produced its ‘Mathematics and IT – a pupil’s entitlement’ leaflet in 1995. It lists six major ways in which ICT can provide opportunities for students learning mathematics:

Learning from feedback: The computer often provides fast and reliable feedback which is non-judgemental and impartial. This can encourage students to make their own conjectures and to test out and modify their ideas.

Observing patterns: The speed of computers and calculators enables students to produce many examples when exploring mathematical problems. This supports their observation of patterns and the making and justifying of generalisations.

Seeing connections: The computer enables formulae, tables of numbers and graphs to be linked readily. Changing one representation and seeing changes in the others helps students to understand connections between them.

Working with dynamic images: Students can use computers to manipulate diagrams dynamically. This encourages them to visualise the geometry as they generate their own mental images.

Exploring data: Computers enable students to work with real data which can be represented in a variety of ways. This supports interpretation and analysis.

‘Teaching’ the computer: When students design an algorithm (a set of instructions) to make a computer achieve a particular result, they are compelled to express their commands unambiguously and in the correct order; they make their thinking explicit as they refine their ideas.

These opportunities are equally valid now, but we have more widely available tools to help us provide them for our students. Quite soon there will be a range of resources (activities, worksheets and case studies) provided by QCA on the BECTa practitioners web-site to help teachers map these opportunities onto the main types of ICT resources such as graph-plotters, spreadsheets, dynamic geometry software, computer algebra, small software, Logo, statistics packages, graphical calculators, data-loggers and web-based resources.

As an example we can look a different approach to quadratic functions, their roots and factorisation which illustrates the “Seeing connections” opportunity above. This could be approached using graphical calculators, graph-plotters or integrated software. The idea is taken from page 33 of the `Secondary Mathematics with a Graphic Calculator’ book from the NCET’s ‘The I.T. Maths Pack’ published in 1994.
Can you explain why the curve cuts the axes at the same points as each of the lines? Why is the curve negative between these two `roots', and positive on either side? Can you enter a quadratic expression for $X$ in $Y_4$ which gives an identical curve to the one in $Y_3$?

A specific example of the kinds of changes in teaching approach which may be needed is provided by the Royal Society/JMC report `Teaching and learning geometry 11-19’. The following extract is taken from section 5: the development of the curriculum:

*The revision of the National Curriculum by QCA in 1999 gave the opportunity for greater exemplification of the ways in which ICT impacts on many subjects and their teaching. Yet there is very little specific reference to the use of ICT in the mathematics National Curriculum in general, and in geometry in particular. Geometrical software is now widely used in, for example, engineering and design. Through government and commercial initiatives many secondary schools and colleges have acquired powerful CADCAM packages for use in teaching Design and Technology. By contrast relatively few schools have access to software for teaching geometry in mathematics. Yet by using such software in appropriate ways, pupils can apply their ICT skills to increase their knowledge and understanding of geometry. The software also provides them with the opportunity to acquire and practice geometric skills. Opportunities occur when pupils create, analyse and interpret dynamic spatial images; make and test conjectures about geometrical relationships that they can manipulate; and record and present the results of their investigations.

As with any approach to teaching, the educational use of ICT needs to be well thought through and carefully planned. The TTA has produced documentation to accompany the current programme of lottery funded ICT training for all teachers in which it emphasises the importance of a critical approach to the use of ICT. This expects teachers to know where, when and how to apply ICT to enhance the teaching and learning of their subjects. This advice is particularly important in geometry where a variety of approaches are needed including mental, practical, and ICT enhanced work. Increasingly powerful software is becoming available in education, such as for simulations in science and geography, much of which relies on sophisticated mathematical algorithms. Pupils and teachers in all subjects need to be cautious about accepting computer produced results without question, and mathematics is probably the subject best placed in the curriculum to engender a critical approach. In teaching geometry, caution is particularly needed to avoid making assertions based solely on computational illustrations.

Thus the working group would like to see further development of the curriculum with respect to ... the use of ICT. .. We recognise that this will have implications for resources, materials, assessment and teachers' professional development, as will the effective teaching of proof, modelling, problem-solving and other aspects of geometry. In many respects we need develop a completely new pedagogy in geometry.

Here is an example of dynamic geometry software used to explore the relationship between the areas of quadrilaterals and the areas of the figures formed by joining their midpoints.
The dynamic geometry software tells us that the two areas appear to be equal, however we
drag $A, B, C$ or $D$ about. So we can formulate a conjecture that the area of $EFGH$ is always
half that of $ABCD$. But lots of pictures drawn with ICT do not, of course, constitute a proof –
only very strong circumstantial evidence! Noting relationships, like $EFGH$ always appears
to be a parallelogram, or adding constructs to the diagram, such as a diagonal like $BD$, might
suggest geometric relationships from which we could arrive at a convincing proof.

1.3 Impact on the curriculum

As we know, the mathematics curriculum can take a very long time to respond to
technological changes, such as the development of the electronic calculator. Yet now ICT is
so well established in all branches of industry, commerce and research as a mathematical and
statistical modelling and problem-solving tool there are few people who still use pencil-and-
paper techniques. So while the impact may be slow, we should at least be prepared to re-
examine our own ideas of what are the fundamentally important elements of the curriculum.

To quote again from ‘Mathematics and IT – a pupil’s entitlement’:

As the technology progresses and becomes more prevalent, teachers will also need to
be continually reconsidering the mathematical content of their teaching. Having software
which can, for example, solve systems of equations at the touch of a button has strong
implications for the way particular topics are approached.

Certain topics, such as matrices and complex numbers, have all but disappeared from the
mathematics curriculum. This is ironic since, for example, most graphical calculators will
perform the standard arithmetic operations such as matrix inversion or complex
multiplication. Using these facilities we could now concentrate on deepening an
understanding of what matrices and complex numbers are, and can be used for, without the
need to perform routine and unenlightening manipulations. Given the widespread use of
matrices and complex numbers in engineering, science and other numerate disciplines it
would be good if we could harness ICT to help students understand their significance.

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\begin{bmatrix}
2 & 4 & 1 \\
3 & 2 & 0 \\
1 & 3 & 1 \\
\end{bmatrix}
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\begin{bmatrix}
1.4 & .2 & -.4 \\
-5.2 & .4 & 6.6 \\
\end{bmatrix}
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(2+3i)(3-4i)
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\frac{2+3i}{3-4i}
\]

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e^{i\pi}
\]

One important aspect of the widespread availability of mathematical ICT tools, and easy
access to sources of mathematical information via the Internet, is that they can enable you to
keep in touch with developments in your own subject. While it is the norm for teachers of
many subjects to keep themselves up-to-date with their subject, this does not appear always
to be the case in mathematics. So maybe you could help lead the way by creating a
mathematics learning community and ethos in your own school – and hopefully inspire more
students to want to study, and perhaps teach, the subject?
1.4 Managing the needs of the mathematically more able through ICT based enrichment activities

There are three terms that are closely associated with enrichment and, whilst they are not entirely mutually exclusive, they do represent different agendas.

- **Acceleration**: takes pupils into areas of the curriculum normally covered by older children often resulting in early entry to public examinations. This is sometimes achieved by moving pupils into higher year groups for all subjects or just the subject in which they excel. Alternatively, children can be accelerated within their own class working independently, often with some additional support.
- **Extension**: involves moving outside the syllabus and looking at aspects of mathematics not normally covered within the “normal” curriculum
- **Enrichment**: is about extending pupils’ understanding of the mathematical ideas they have already met by applying them to other situations and problems, often requiring decisions on what area or areas of mathematics to employ. The aim is to develop higher level problem solving and communication skills. It extends the notion of using and applying. The aim is to produce a thinking mathematician who can look beyond the standard “test” type questions.

More able pupils need to be stretched and motivated but this is often very difficult in a classroom with 30 or more pupils all needing individual attention. One of the most powerful resources available to you is the computer. More able pupils will no more benefit from the computer than other students but in the same way as ICT can support basic literacy or numeracy skills through targeted, skills focussed, software or the use of generic software (such as word processors and spreadsheets) or access to resources via the Internet, it can support the needs of the more able mathematicians. Effective use of ICT will enable you to:

- find appropriate online resources that can be used off-line or converted to a paper based resource – an example of this are the resources to be found on the NRICH website (offering enrichment materials for mathematics to pupils of all ages)
- use generic software and open ended tasks that will engage pupils who are struggling but offer real opportunities for stretching the most able. Software such as Logo, dynamic geometry software and spreadsheets all have this potential but closed, skills based, software does not give the flexibility to encourage enrichment.

This can be achieved by:

- Planning tasks for the whole group that will involve all pupils but stretch the most able (differentiation by outcome). So, for example the whole group might be using Logo to find their way through a maze – working on right and left and estimating distance. At the same time in the same room more able pupils may be creating a maze of their own. What are the properties of mazes? How can you ensure that the width of passages remain constant? This will involve using the same mathematical ideas but extending them to more demanding contexts. What they are not doing is “more of the same”.
- Sending individual or small groups of pupils to work at a computer on tasks particular to their ability but related to the activities being undertaken by the rest of the class. So if the group are working on angle properties of 2D shapes a small group might be investigating the angle properties by creating tiling patterns in Logo.
- Using the Internet to locate resources, such as those from NRICH to provide enrichment material that supports the lesson objectives and the more able pupils.
So where might you find ideas for appropriate activities? Some useful sites that might provide some starting points for enrichment activities:

- **NRICH** [http://www.nrich.maths.org](http://www.nrich.maths.org)
- **Games by Conway** [http://www.cs.uidaho.edu/~casey931/conway/games.html](http://www.cs.uidaho.edu/~casey931/conway/games.html)
- **Komal** [http://komal.elte.hu/info/bemutatkozas.e.shtml](http://komal.elte.hu/info/bemutatkozas.e.shtml)
- **Abacus** [http://www.gcschool.org/abacus.html](http://www.gcschool.org/abacus.html)
- **Mazes** [http://www.logicmazes.com/index.html](http://www.logicmazes.com/index.html)
- **Fibonacci** [http://www.ee.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html](http://www.ee.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html)

### 1.5 Implications for Management

Inevitably you will become involved with management – not only of deploying ICT in your own teaching. You may be responsible for managing others, and you may need to persuade your own managers to support you with your own, or the department’s, development plans.

The role of school management at any level is to develop, support, and direct the learning community. For the purposes of this document we will focus on the management aim of developing, and supporting, a culture of creative use of ICT within the learning and teaching of mathematics, where classroom mathematics, at its best, is understood as a social activity fostering a community of enquiry.

Two general features of ICT applications that enhance the learning experience are software that is **responsive** and techniques that provide **leverage**, making easy something that was otherwise a considerable effort. We will try and give a flavour of what we mean.

**Responsive software** – examples include where a change to one value on a spreadsheet automatically changes all calculations that depend on that as a source value, and where movement of one point on a dynamic geometry display causes movement in all constructions that depend upon that point.

It is useful to make explicit the value of “interactivity”, i.e. responsiveness, in software.

- Interactive software encourages learning by experiment and discovery – the “try it and see what it does” and “why did that happen?” approach.
- A facility to make changes, with instant effect, invites generalisation.

**Leverage** - making easy something that was otherwise a considerable effort. Examples of leverage include moving text around within a word processor document, replication of formulae on a spreadsheet and the automatic production of graphs on a graphical calculator. Any of these would have been tedious, or even worthless, without ICT.

There is a proposed ICT Development Cycle with phases which managers need to support.

The **familiarisation** phase applies both to students and teachers – this is a period of orientation and uncertainty. During the familiarisation period an important initial perspective forms about what the new object is, and what it might be worth, together with an assessment about what is being threatened. The familiarisation phase is important and needs to be handled by managers with awareness, imagination, and clear purpose. The familiarisation phase is passed through many times as a teacher or pupil moves on to encounter each major new environment. The teacher who, for example, has used graphical
calculators in the classroom for some time, but is coming to a dynamic geometry package for
the first time, will experience again the excitement, uncertainty, and perhaps frustration, of
the familiarisation phase.

In managing the familiarisation phase, the context within which the initial encounter occurs
matters greatly. The initiation experience needs not only to reveal just what the tool does,
and give enough practice to sow the seeds of confident use, but also to convince the new user
that the tool has value. For example, the first use of a spreadsheet should be based on
something mathematically interesting, such as a number puzzle like the magic square.

This is followed by a phase known as ‘Routine Specific Activity’, which is needed before a
general culture of creative use establishes itself. In this period of routine but specific use,
particular lessons will involve particular activities which make use of particular techniques.
The management of routine specific activity includes the provision of schemes of work, the
production of supporting resources, the provision of classroom support, and, of course,
provision of access to hardware.

Barbara Ball, the ATM’s Professional Officer, contributed to the review ‘Developing IT
Across the Mathematics Department’ published in 1996. Here she compares her recent
experiences with ICT tools to those some 6 years previously.

I think I have found both Geometer’s Sketchpad and Cabri relatively easy to learn
because essentially what each of them does is to simulate ruler-and-compass constructions
in a straightforward way. Of course, I needed to learn which menus to look in and which
buttons to press. But once I had found my way round that I could do almost anything with
the software and I could easily switch between the two programs. This is because I know
about ruler-and-compass constructions. Of course, if someone does not know about ruler-
and-compass constructions it might be difficult for them to do very much at all.

Pinpoint was different. In order to use Pinpoint I had to learn, not only which
buttons to press, but also how the program author was choosing to interpret and organise
data and how I was expected to present the data to the parts of the program that draw
graphs and charts and how the program author interpreted the meaning of different graphs
and charts. I suppose this could be summarised by saying that different people organise
information, and interpret the meaning of words such as ‘ histogram’, in slightly different
ways. There are no right answers about this; it is largely a matter of convention. In
contrast, there is only one ruler and one pair of compasses and either we know how to use
them or we don’t.

In order to establish the mathematics classroom as a community of enquiry there are issues
for management. These include the provision of resources and CPD for teachers in order to
ensure all students have equal opportunity to experience mathematics using ICT. At a senior
level this could involve a clear development plan to meet national targets for the proportion
of lessons using ICT. It also involves ensuring that the knowledge base is kept up-to-date in
order, for example, to make informed decisions about the relative strengths of different ICT
provision, such as computers with interactive white boards as against hand-held technology.

There are management issues for the development of ICT in mathematics at all level.
Appraisal by middle managers can be used to identify needs in a supportive way and also to
relate these to training needs. Middle managers have a pivotal position in facilitating
teachers’ development and in ensuring students have a good experience of ICT in
mathematics.
Appraisal can provide a useful means to identify needs, provide support and monitor the professional development of teachers in the use of ICT. In the best management practice this supports the whole school and departmental development plan. Our experience is that teachers respond positively and find these aspects of the appraisal and target setting process constructive.

One examples of good practice is to identify key modules in schemes of work which will require all mathematics teachers to use ICT in an aspect, say, of the geometry curriculum. Planning this, and finding department time for training, means that teachers who need support can get it, but at the same time they are challenged to deliver lessons with ICT. Another successful strategy has been to hold mathematics enrichment days for a particular year group which involve the use of ICT.

1.6 Evaluating the impact of the use of ICT

Evaluation occurs at many levels – e.g. of the use of some ICT as part of a lesson, or of the lesson as a whole, or maybe of your own, or the department’s, facility with ICT. You will also need to be able to evaluate particular ICT tools in considering their usefulness for mathematics teaching. Let’s start with how you might evaluate a lesson which used ICT.

Evaluating a lesson using ICT

Useful references here might be the ‘Review and Evaluation’ questions on page 194 and the ‘Planning a lesson’ on page 222 of `Teaching Mathematics with ICT’. Also the list of bullet points on page 9 of the TTA’s `Identification of Training Needs – Secondary Mathematics’.

A first level evaluation might just be an identification of:

- What are you trying to do?
- How are you trying to do it?
- Who are you going to work with?

A second level might extend the questions to:

- Did you meet the learning objectives?
- Were the students actively engaged and were all abilities catered for?
- Were the students confident?
- Did the plenary work – could they transfer knowledge to other scenarios?
- What mathematics did they use?
- What progress did they make?
- Can it be extended further?

The Oldknow & Taylor book includes a sample activity review sheet. Below is a reproduction of a teacher’s evaluation of a lesson exploring number sequences with graphical calculators:
1. What did you expect to get from the task, i.e. purpose and learning intentions?
These are defined in the Teaching/learning objectives in the lesson plan.

2. What additional knowledge and skills did you need:
   about the technology
   Using the data handling facilities associated with the "stat" key and the calculator's ability to manipulate "lists". For the follow up lesson: how to plot data and superimpose functions.

   about the mathematics
   Fairly confident with the mathematics, but talked to mentor about the expectations and level of challenge appropriate for this Year 9 group.

   about teaching strategies and approaches?
   How to engage all pupils in the whole class interaction, use of the growing design and matchsticks as an initial task accessible to all pupils. The use of more open-ended questioning at the start of the lesson to establish what pupils already know. Getting the pupils to discuss in pairs to ensure everyone is involved was a good idea. I like collaborative work and often use this approach in my lessons. Pupils learn so much when they have to explain their thinking to others. It also improves their listening skills.

3. What additional knowledge and skills did the pupils need:
   about the technology
   How to enter, edit and manipulate data in lists.

   about the mathematics
   An ability to analyse growing patterns and convert observations in words into conventional mathematical symbols

   about learning strategies and approaches?
   Be prepared to explore, look for patterns and try things out, develop their use of visual imagery to explain their thinking. Work and co-operate with other pupils in their group. Explain and communicate ideas to other pupils in the class.

4. Was the focus of your teaching on developing skills or understanding?
   Both; I wanted pupils to develop pupils reasoning and communication skills through an accessible context. I also wanted to develop pupils understanding of equivalent algebraic expressions and introduce them to algebraic manipulation and give purpose for further work on this topic. Although I think it is much more important that pupils can use algebra to formulate/model situations.

5. Did the pupils focus on understanding or pressing buttons?
   At the beginning of the first lesson some time was spent getting used to the data handling facilities on the GC. However, pupils were quick to come to terms with these and after an initial hiatus were able to use the GC with confidence to explore, check and validate their algebraic rules.

6. In what ways were your answers to questions 4 and 5 affected by the use of the technology for the topic?
The technology acted as an extra "teacher". Pupils were able to try out their ideas on the calculator and it provided them with instant feedback on the validity of their conjectures and formulae.

7. **Would the use of the technology for this topic change the order in which concepts were taught?**
   Might try to introduce simple modelling and formal algebraic conventions earlier. Pupils coped well with the symbolism in the context of growing geometric patterns and had little difficulty using the GC.

8. **What were the benefits/disadvantages of using the technology?**
   Benefits; see 5, 6 and 7. Disadvantages; some initial problems getting to know the GC. I had considered using "mouseplotter" and/or a spreadsheet in the school's computer room but chose the GC because of its easy use in the classroom and the similarity of the GC's notation with the standard algebraic convention.

9. **What would you do differently next time?**
   As pupils become familiar with the GC I will get the pupils to plot the results and link the graphs to the rules such as the kind of "rule that produces a straight line and the connection with the gradient and the intercept.

**Evaluating the ICT use of the mathematics department**

Part of the work of the DfES’ mathematics curriculum IT support group was to produce materials to help departments evaluate their use of ICT and their future needs. A booklet giving case studies of schools called ‘Developing IT Across the Mathematics Department’ is available on-line at: [http://curriculum.becta.org.uk/docservlet.php?docid=1892](http://curriculum.becta.org.uk/docservlet.php?docid=1892)

*This booklet ... is designed to help schools monitor their progress and form their action plans. It is based on case studies of six schools. These are all very different and there is no single clear route which they all advocate, but the practices in each school are outlined and policies for the use of IT in mathematics are pursued. In addition to the descriptions of the schools, this book contains discussions of some important issues common to the process of review:*

- Staff development
- Deployment of resources
- Linking practice with Entitlement opportunities
- Departmental policy.


The review is divided into 7 sections:

(this has been updated slightly, for instance where the original review form referred to IT we have updated this to ICT)
1. ICT in learning mathematics

This section establishes the extent to which ICT is built into the teaching of mathematics in all classes. In which years are there modules of work including ICT use, support materials and teaching notes? How do we use the technology as a teaching and learning resource?

Year 7
Year 8
Year 9
Year 10
Year 11
Years 12, 13

What aspects of ICT do we emphasise?

2. Current ICT skills of mathematics teaching staff

This section establishes the types of software, and devices, with which teachers are familiar and how widespread ICT skills are across the department. Since the development of all these skills takes time, the first part is about continuing professional development (CPD). Recent ICT/mathematics CPD activities for the department or for individuals:

Number of teachers who teach mathematics: 
Number of full-time equivalents in mathematics department:

Number of teachers who have developed specific ICT skills e.g:
Data-handling used database or statistical software:
Dynamic geometry used DGS for constructions and transformations:
Graph-plotters used graph plotting software in their teaching:
Graphical calculators used numeric and graphing aspects of graphical calculators:
Logo used Logo to manipulate a screen turtle and write procedures:
Small software used curriculum focused software e.g. Smile, Maths Alive:
Spreadsheets set up a basic spreadsheet e.g. entered and replicated formulae:
Symbol manipulation used computer algebra software:
Web-sites explored web-sites relevant to mathematics teaching:
Whiteboards used electronic whiteboards for whole-class work:

Other relevant ICT skills:
3. Curriculum resources
This section establishes whether there are appropriate mathematical activities and other support materials (published, or produced within school) for the different categories of ICT: Departmental resources/starting points for working with:

- Data-handling
- Data-loggers
- Dynamic geometry
- Graph-plotters
- Graphical calculators
- Logo
- Small software
- Spreadsheets
- Symbol manipulation
- Web-sites
- Whiteboards

Other relevant ICT skills:

4. ICT resources
This section is about the hardware available in the school. How many machines (desktop PCs, laptops, palmtops, graphical calculators etc.) are available in mathematics classrooms? Comment on suitability, sufficiency, accessibility and reliability.

How many purpose equipped computer suites are there; how many are bookable for maths?

Comment on the adequacy, or otherwise, of the bookable resources:

Time available for maths classes:

Availability and suitability of generic software (e.g. spreadsheets):

Availability and suitability of mathematics software:
5. Entitlement opportunities:
`Mathematics and IT – a Pupil’s Entitlement’ highlights six major opportunities for exploiting the power of the computer. This section establishes which opportunities are offered in your school. Indicate on a scale from 1 (minimal) to 5 (extensive) the extent to which pupils in your school have access to the following opportunities:

Learning from feedback:

Observing patterns:

Seeing connections:

Working with dynamic images:

Exploring data:

Teaching the computer:

What do you believe are the most important benefits which using ICT brings to mathematics teaching?

6. ICT development
This section establishes how you arrived at your current stage of development.
Can you identify helpful factors? Have you had contact with other schools through an LEA group, subject association, project etc.? Are ICT resources available for teachers to take home? Can you identify helpful sources of ICT support and advice inside the department, school or elsewhere?

7. Action plan
Completing the previous sections may have suggested directions for development. Your action plan could be written under the following headings, giving some indication of time scales.
Embedding ICT (entitlement opportunities and curriculum objectives):

Staff development, both within and outside school:

Updating and improving ICT resources:

What is needed in order to achieve this plan within the stated times?
During 2002 BECTa has also been developing materials to support secondary school subject departments evaluate their work. The mathematics materials are to be published on BECTa’s ICT advice site http://www.ictadvice.org.uk under the title: `Timesaver: Carrying out an ICT audit of your Mathematics department’.

Evaluating resources such as software and web-sites.

The BECTa Educational Software Database http://besd.becta.ork.uk/ contains some useful information about ICT resources, which can be searched by subject, age-range, or both.

It gives details about, and links to, suppliers including costs of licences etc. In some cases it links to evaluations provided by teachers contributed as part of a group called TEEM, standing for Teachers Evaluating Educational Multimedia http://www.teem.org.uk/.

The evaluations are conducted to a common framework and use headings such as:

Overview of teaching with this title:
Installation:
Content
Curriculum relevance
Design and navigation
Ease of use
Special needs
Courseware
Conclusion

For an example see the review of the DLK MathsWork CD-ROM for KS3/4 contributed by Peter Garner of Blackheath High School: http://www.teem.org.uk/findcdorweb/cdtitles/fulleval?cid=72A&rid=001&eid=e&pid=G003
Section 2: Review of ICT tools of particular relevance to mathematics education

Here we aim to provide practical advice about specific resources for you to consider when planning ICT developments in your own teaching, or for your department. This includes both software and hardware. As with many aspects of ICT, it is a sensible starting point to see what advice is already available and then to bring it as up-to-date as possible.

During 1999 BECTa held a number of seminars to review the software base in relation to subjects in the secondary school curriculum. The mathematics report can be found at: http://www.becta.org.uk/technology/software/curriculum/reports/maths.html

The mathematics report identifies a `toolkit' approach structured around a number of categories of software. This also forms the basis of ICT guidance given in the Key Stage 3 Mathematics Strategy’s `Framework for teaching mathematics: Years 7,8 and 9’, page 25. A different approach is adopted in the TTA’s ‘The Use of ICT in Subject Teaching: Identification of Training Needs: Secondary Mathematics’. On page 9 there is a very useful analysis of aspects of KS3/4 content where teaching and learning can be enhanced with ICT, as well as examples of ICT use to support the teacher.

We have taken this opportunity to update this useful information base, and to restructure the categories of both software and other ICT products. In order to make this as useful as possible we will refer to specific products which the group has experience of using in a mathematical context. Inevitably this cannot be comprehensive, nor stay up-to-date, but we hope that it will give you a clear idea of the extensive, and not particularly expensive, range of ICT tools already “out there” waiting for you to experiment with.

The DfES’s Year 7 RM MathsALIVE! project aimed to ensure that its pilot schools were equipped with just such a `toolkit’. This project has now been developed into the full Key Stage 3 Maths Alive Framework Edition service available from Research Machines: http://www.rm.com/Secondary/Products/Product.asp?cref=PD7370

One of the main lessons from that project is the benefit which comes from selecting a range of tools which are (a) not too many in number and (b) have widespread uses across the whole mathematics curriculum 11-19. This way the time you spend learning to use a tool is amply repaid when it can be used throughout the classes which you teach. It allows the department to share out responsibility for taking a lead role with a particular tool.

The main tools used in the project were:

- Whole class displays: Smartboard interactive whiteboards with data projectors and PCs using RM’s `Easiteach Maths’ software - and also Texas Instruments’ graphical calculators with LCD pads projected using OHPs.
- Hand-held technology: TI graphical calculators and data-loggers.
- ‘Small programs’: addressing particular aspects of the curriculum – mostly in the form of games and simulations written by RM’s 3T production company.
- Programming languages: `MSW Logo’
- General purpose software: Microsoft `Office’ products such as `Excel’, `Word’, `Explorer’ and `Powerpoint’
- Mathematics teaching software: `The Geometer’s Sketchpad, V.4’ dynamic geometry software and the `TI Interactive!’ integrated mathematics package – including graph-plotting, data-handling, symbolic algebra etc.
In this section we review a range of ICT tools. An important maxim to bear in mind, especially when considering the hardware (sometimes known as `platforms’) you might use, is to **try to keep as much flexibility as possible in the deployment of ICT** – it should be adapted to the way you conduct mathematics, and not the reverse!

### 2.1 Whole class displays

The TTA’s Needs Identification CD-ROM pack for secondary school has four case studies on the mathematics disk. These each illustrate a different means of obtaining a whole class display. Technology has moved so fast that when these were produced, just three years ago, it was considered too ‘leading edge’ to illustrate the use of an interactive whiteboard! For an extensive review of whole class display technology see the BECTa web-sites: [http://www.becta.org.uk/teaching/pedagogy/technologies/whiteboards/html](http://www.becta.org.uk/teaching/pedagogy/technologies/whiteboards/html) and [http://www.becta.org.uk/technology/infosheets/html/largescreens.html](http://www.becta.org.uk/technology/infosheets/html/largescreens.html).

These also include an extensive list of suppliers. As the information goes out of date very quickly you will need to check what products are currently available, as well as the prices!

#### Digital data projectors

These projectors usually accept a range of signals from external devices such as:

- computers, using a standard graphical output format such as SVGA, XVGA etc.;
- video recorders and players, cassette and DVD, using a standard format such as PAL;
- other electronic display and capture devices, such as digital cameras and camcorders.

They can be permanently mounted e.g. from the ceiling (as in the photograph on page 7), or put on a stand or table. While still relatively expensive items, their price has fallen sharply in the last couple of years and portable versions are available from about £1500. The major part of the cost is the lamp which has to produce a very bright image indeed. Because of the heat generated, the unit includes a cooling system. This makes it imperative not to move the unit until the lamp has been sufficiently cooled after it is switched off. The increasing home use of data projectors, e.g. for showing movies from DVD, means that prices are likely to continue to fall, and that a good range of units are likely to be on the market. It also means that projectors have become more attractive to thieves and that security is an issue to be considered. The image can be projected on any flat, light surface – but preferably one which is not reflective – so an ordinary classroom whiteboard may not be ideal. The Geometry case study on the TTA CD-ROM illustrates the use of a data-projector.

#### Interactive whiteboards

An ’Interactive WhiteBoard’ (IWB) is one part of a system which includes a computer and a data projector. It is both an input, and an output, device which allows information to be sent to, and received from, the computer. There are two main types of IWB. The ‘digital’ version uses a special stylus or electronic ‘pen’. The ‘analog’ version responds to any touch, such as with a finger. In either case the IWB sends back information to the computer about exactly where the pointing tool is placed. An IWB comes with supporting software which can allow screens which have been created to be saved and printed. It also allows the pointing tool to be used to annotate the output from other software. A matter of cognitive
importance is that you click at the point of action on the display screen rather than via a mouse alongside a PC’s keyboard or with a tracker-ball or touch pad built-in to a laptop.

The use of an IWB can be considered at several levels:

- **low interactivity**: for example you move from one slide to the next in a `Powerpoint` presentation by clicking on an on-screen icon;
- **medium interactivity**: for example where teachers and students are able to control relevant software from the front of the class for whole class teaching and discussion;
- **high interactivity**: for example where students are able to interact with geometrical constructions produced using dynamic geometry software by dragging diagrams into different configurations – the evaluation of the Y7 project referred to this as “pupils touching the mathematics”.

In addition to the main display screen – usually wall-mounted or stand-mounted – and its pointing device(s), you can purchase other external input devices, such as modern electronic versions of the Victorian school slate which can be passed around the class. There are some routine maintenance procedures you need to adopt to ensure that the PC keeps track reliably of the position of the pointing device. IWBs come with software to enable you to recalibrate the board by clicking on a number of crosses displayed on the board. This needs to be carried out periodically even if you are using a fixed projector.

Sources of IWBs include:

- Promethean Activ Board [http://www.promethean.co.uk/index.html](http://www.promethean.co.uk/index.html)
- Smartboard [http://www.smartboard.co.uk/](http://www.smartboard.co.uk/)

A review of IWBs can be found in `Teaching ICT` Vol 1, issue 2, Autumn 2001 (This is the journal of ACITT: the association for ICT in Education). A typical large board (60” to 75” diagonal) costs around £1500, but you will also need to budget for additional items such as a data projector, stands and/or fixing costs.

One of our contributors provided the following comments:

> These comments follow my personal full-time use of an IWB in my classroom for the last six months. There is no doubt that the system represents a considerable advantage over just a plain whiteboard, or over a digital projector and static screen (which I was using before). The learning curve for the user is not steep: at the lowest level of competence you can just use the system as a conventional whiteboard, but with the advantage of being able to review material that would otherwise have to be wiped off due to lack of space. Graphics and web links can be prepared beforehand and a lesson’s `scribbling’ can be saved and/or printed out.

> One major use is the ability to annotate over other packages. So, if you’re working with a dynamic geometry document or a spreadsheet, you can simply switch to annotation mode and write over the top of the document window. The ability, with a single pen, to quickly switch between different colours, highlighters, and specialist drawing tools, adds significantly to the quality of presentations.

> I’ll end by describing one of the flipcharts that I have produced for work with students on an AS Pure Maths course. The flipchart consists of a number of pages: on each page is the graph of a function and the graph of its derivative. Students must decide which curve is the original function, and which is its derivative. The activity encourages thinking
and verbalisation of ideas about gradients. The task can be made harder through the choice of function and omitting axes and scale.

TV/video output

A cheaper solution than a projector (with or without IWB) is to echo the computer display onto one or more large screen TV sets. This requires a different form of output (PAL video) than that sent to an external monitor (usually a variety of VGA, such as SVGA or XGVA). Some computers and laptops have such video output built-in. Otherwise you will need a ‘scan-convertor’ which takes a VGA input and produces a video output. An example is the Lindy http://www.lindy.co.uk VGA convertor Pro at c £165. The Data-handling case study on the TTA CD-ROM illustrated the use of a VGA/TV convertor and TV display.

OHP pads

There are a number of devices using Liquid Crystals Displays (LCDs) designed for use with an overhead projector. These include:

- transparent four-function and scientific calculators which sit on the OHP
- monochrome LCD displays which echo the output from graphical calculators
- colour LCD displays which echo the output from a PC or laptop.

In each case the OHP needs to be of the conventional type with its bulb in the base, rather than the portable version with a bulb at the top reflected through a mirror in the base. While the first two of these devices work quite well in normal lighting conditions, the third option usually requires some form of black-out, and works best on an OHP with a particularly powerful lamp. On the TTA CD-ROM, the Algebra case study illustrates the second option and the Number case study illustrates the third.

2.2 Hand-held technology

Calculators

While these are far from leading edge technology, there are still several issues concerning their use which need to be borne in mind. Clearly it is important that pupils know how to use them, and also that they recognise when it is sensible to use them. The use of calculators is required in Key Stage 2 mathematics. They are used in many subjects of the curriculum, and advice about their use is provided in KS3 Mathematics Strategy’s ‘Numeracy Across the Curriculum’ materials. OHP versions of calculators can be useful in supporting whole class teaching. Another approach is to display the simulations of basic and scientific calculators provided in the MS Windows accessories! You do need to know how your calculator works.
There are two major types of calculator operating systems, arithmetic and algebraic, the latter tending to be slightly more expensive. Note that they can give different answers e.g. to $2 + 3 \times 4$. What would you need to do to ensure you get the same answers? Where would you predict other differences? Algebraic OS calculators will provide correct solutions to calculations without need for rearrangement. They also have a useful constant memory function which can be useful for routine conversions e.g. £ to euros.

**Graphical Calculators**

It is convenient to use the label ‘graphical calculator’, or the abbreviation GC, to refer to a hand-held device which has all the features of a calculator together with an LCD display screen with built-in graph-plotting software. But in the 15 years or so since these first came on the market they have developed greatly and there are considerable differences between models, just as there are between PCs! There is very useful advice, together with good links and references, on the BECTa web-site at:


In this country the use of GCs at A-level was investigated and developed by the MESU project ‘Graphical Calculators at A-level’ directed by Ken Ruthven and at KS3/4 through the NCET ‘Portable Computers Pilot Evaluation’. Their use is now common in KS3/4 mathematics and has been considerably reinforced by the large numbers of references in the KS3 strategy’s ‘Framework for mathematics’. At A/AS-level there is a specific assessment objective concerned with their use, and many of the Free Standing Mathematics Qualifications have a specific requirement to use them. In many other countries (e.g. Netherlands, Denmark, Portugal) it is compulsory to teach the use of GCs in secondary schools. Their use is strongly encouraged in national examinations at 16-19, such as the US Advanced Placement examinations and the Scottish Highers. They are compulsory in 3 of the 4 mathematical strands in the International Baccalaureate, and strongly recommended in the fourth.

An example of their use is provided in Section 3: ‘Using ICT in secondary mathematics – a case study’ (pp 34-47) in the TTA’s “Identification of Training Needs: Secondary Mathematics” from which the following is extracted:
Specific teaching points
Now that the pupils were familiar with the problem the teacher reminded them how to enter a function such as "$Y_1 = X\sqrt{(5^2-X^2)}$" on the graphic calculators and show the class how to use them to produce automatically a table of values of the areas $Y_1$ for the values as before for the heights $X$, as in Fig. 4a. (She would expect pupils to explain why the word "ERROR" appeared against the value $X=6$.)

Specific teaching points
The teacher checked that pupils knew how to decide upon suitable values for the scales on the X- and Y-axes based on the data in their tables: "What's the smallest value $X$ can be? What's the largest...." She checked that pupils remembered how to trace along the graph. She also reminded them how to "zoom in" on places of interest on the graph, and also how to arrange the scale on the x-axis so that each sideways movement of the cursor corresponded to a "round number" for X (as in Figs 5 a, b, c).

Step 5 - drawing graphs of functions
The teacher now discussed with the class how the data might be represented differently and more accurately using a graph. She then checked whether the pupils remembered how to use the graphic calculators to draw the graph of the function stored in $Y_1$.

The teacher worked through a complete example with the class using the values in Fig. 4a as co-ordinates for the graph.

Commentary on teacher’s decisions about whether or not to use ICT in meeting the teaching and learning objectives and justifying and explaining any use of ICT
The teacher judged that graphical calculators were the best resource for this as pupils could produce graphs quickly using the data. The split screen facility which allowed pupils to view the graph with the table of results alongside it, gave the teacher the opportunity to draw out in discussion with pupils the connections between the two representations.
Finally she showed pupils how to arrange the screens of the graphic calculators so that the table and the graph were displayed alongside each other. Now when the graph was traced the pupils could see (a) the point on the graph, (b) the algebraic expression for the function which generates it, (c) the co-ordinates of the point, and (d) the corresponding entry in the table (Fig. 6).

![Fig. 6. Split screen display](image)

The same case study was used as the basis for the Year 10 Algebra example on the accompanying TTA CD-ROM. This shows both pupils working in pairs at the problem using GCs, and a teacher’s GC being used by both teacher and students for whole class displays.

At a basic level a graphical calculator is a small computer running built-in software specifically designed for mathematical, statistical and financial applications and displayed on a low-resolution monochrome screen. In that respect there is very little difference between the various models from the main manufacturers: Casio, Hewlett Packard, Sharp and Texas Instruments. As with the purchase of any ICT, the maxim is: **you get what you pay for**! For around 50% more than the cost of a basic model you can purchase models which are designed for easy transfer of data, screens and programs between calculators, PCs and compatible devices such as data-loggers and even robots, as well as for projection using an OHP or a video input. The large amounts of so-called flash-ROM now available in such models mean that you now have the equivalent of a mini hard-drive on which to store application programs. Such applications can be pre-loaded, or obtained via the Internet, and cover a wide range of aspects from mathematics (e.g. dynamic geometry), ICT (e.g. spreadsheet), science (e.g. periodic table) etc. A recent BECTa project explored the use of versions of software, such as from SMILE, on graphic calculators. It concluded that:

This short project demonstrated the potential of graphic calculators to enhance the teaching and learning of mathematics throughout the 11-16 curriculum. Both teachers and students were positive about the use of the calculators. Given the difficulty of accessing computer suites in school, graphic calculators offer mathematics teachers a dedicated resource that can be used to target particular areas of the curriculum in a way which encourages mathematical thinking. The outcomes show the flexibility of usage that graphic calculators allow: whole class with one demonstration machine, whole class with individual or paired use, small group or individual use when other activities are also going on in the classroom. There is also tremendous potential for covering a range of mathematics content: number, algebra and data handling.
Other hand-held devices

The range of these is constantly increasing. Personal data assistants (PDAs), palm-tops and new generation mobile phones all have educational potential, although they were not designed for that purpose. Their facilities vary widely, and their use in schools is not established, so it will not be possible to give sound pointers to their use at this stage. However there is one product, based on a “game-boy” design which has been in use in education for a number of years, although mostly outside the UK. This is the Texas Instruments TI-92 which has the features of an advanced graphical calculator, but whose mathematical software includes symbolic manipulation (written by one of the authors of the popular Derive software). The built-in software includes dynamic geometry (either Cabri or Sketchpad). The French government supported a successful trial of this technology used by Lycée teachers with an OHP pad as an electronic blackboard for teaching mathematics 16-19. A new machine, the Voyage 200, using similar technology, but with a large flash-ROM memory has been announced: http://education.ti.com/product/tech/89/features/features.html.

2.3 ‘Small programs’ which address particular aspects of the curriculum

There are many programs currently available which are designed to exemplify particular mathematical topics. Some are games, others are more didactic. They vary widely in quality. A number of titles have been developed by educational groups and trialled in classrooms. These include those produced by SMILE, the ATM and NRICH. Some are available at no cost via the Internet, such as NRICH. Others are commercial products. The RM MathsAlive Framework Edition includes a large number of examples of small programs developed by their 3T production company – you can try some of these for yourself, such as ‘Codebreaker’, from the BECTa site: http://www.becta.org.uk/ks3online/maths/index.html.

An interesting recent development is the availability of versions of such software to run on graphical calculators.

The criteria for using such software should start from the mathematical learning objectives. The programs can be very motivating for pupils, but it is important to consider the quality of the interaction between the pupil and the software. For example, feedback can be crucial.
Sometimes these resources work best with a number of pupils, as a stimulus for discussion and thinking. Some will also work well as resources for working with a whole class.

Examples of such software are included in BECTa’s “Using web-based resources in the daily mathematics lesson”, designed for primary schools [http://www.ictadvice.org.uk](http://www.ictadvice.org.uk). One well-known example is ‘Monty’ which is a program (originally developed by the ATM) that is based on the hundred square which can be arranged in various patterns. These patterns can be partially obscured by a snake (Monty Python!) and the pupils have to predict which numbers have been hidden. Some of the patterns can be very challenging. ‘Monty’ can be used by individual pupils, small groups or by a teacher working with a whole class. The software is freely available on CD-ROM as part of the National Numeracy Strategy’s “Using ICT to support mathematics” pack, available from DfES publications [dfee@prolog.uk.com](mailto:dfee@prolog.uk.com), and can be used on-line at the Standards site: [http://www.standards.dfes.gov.uk/numeracy/](http://www.standards.dfes.gov.uk/numeracy/).

The screens below show the ‘NumCurio’ application for the TI-83 Plus:
2.4 Programming languages

Algorithms, sets of instructions to carry out procedures, play a key role in mathematics. Students encounter a number of algorithms, from long multiplication to Newton-Raphson, which are amongst the major achievements of mathematics. All too often learning such algorithms is a chore, partly because learners are not given the opportunity to experience the power of creating algorithms. “Teaching the computer” is an opportunity for students to gain some insight into what it means to design an algorithm and hence to begin to appreciate the importance of algorithms.

There are a variety of ways of providing opportunities for students to “teach the computer”. Two very useful ones are with Logo and with graphical calculators.

**Logo**

*Logo* has a graphical interface which students use to instruct an on-screen “turtle”. This is a good place to start. Giving pupils a few turtle instructions, such as Forward: FD, Right: RT, Clear Screen: CS, Left: LT, means that a simple question like “Can you draw your initials?” has enormous potential for mathematical discussion and discovery (linked to areas such as estimation of angles and distances and properties of shapes). Alternatively, providing students with the instructions to draw a rectangle can allow students to draw a square by changing some of the instructions. The instructions to draw a square can be saved as a *Logo* procedure and this procedure, in turn, can be modified to take inputs so that the turtle can draw any regular polygon. In this way the turtle has been taught a general algorithm.

```
to Rectangle
  Repeat 2 [FD 100 RT 90 FD 50 RT 90]
end
```

```
to Square :edge
  Repeat 4 [FD :edge RT 90]
end
```

*Logo* draws right and left turns dynamically and emphasises the true nature of angle being an amount of turn. *Logo* is not just a geometrical tool but can be used to investigate number patterns and sequences. There is potential to use *Logo* as a tool for developing an understanding of variables and encouraging mathematical communication. A key feature is that any *Logo* procedure that is created can be used again within more complex procedures. By building such `super-procedures’, students can move on from turtle graphics to computational modelling of probability, iterative processes, etc. Some versions of *Logo* are available free, such as MSW *Logo* (which can be found at [http://www.softronix.com](http://www.softronix.com)). Other useful versions of *Logo* include *StarLogo* and *LogoWriter*.

**Graphical calculators**

All models of graphical calculators are programmable, so that facilities which are not readily available on a particular model can usually be added by writing a small program for the purpose. Their programming language is a variation on BASIC. An interesting account ‘Algebra by stealth – cartoons on graphic calculators’ of programming work with a class of Y8 girls is given by Alison Clark-Jeavons in the report ‘Good Practice in the Use of ICT in Mathematics, Science and Geography at Key Stage 3’ edited by Joyce Wood and obtainable from RM plc: [http://www.rm.com/_RMVirtual/Media/Downloads/good_practice_ICT.pdf](http://www.rm.com/_RMVirtual/Media/Downloads/good_practice_ICT.pdf)
My Year 8 students learn to produce animated pictures on graphic calculators. The purposes of this activity are to develop students’ understanding of co-ordinates, provide a real application of algebra and develop their problem solving skills, by answering the question, “Here is a co-ordinate picture; now how can we make it move?”

The activity uses relatively low-level, inexpensive technology and can be carried out in the classroom. Each student has a hand-held TI-83 graphical calculator and the teacher has a calculator Viewscreen placed on an overhead projector. The Viewscreen facilitates the projection of the teacher’s calculator screen for the whole class to see. It is possible to link a student’s calculator easily with the teacher’s to share an individual student’s work with the whole class.

The first task is to create a co-ordinate picture. To do this, the students input a set of co-ordinates into the statistical list facility on the calculator and set up a statistical plot. The level of difficulty can be varied here, with students entering positive/negative integers or decimals to create a picture in one, two or four quadrants. I use a planning sheet to enable less able students to plan their pictures initially and note the co-ordinates on paper. The feedback given by the calculator display allows students to spot and amend any errors quickly and easily when producing the static picture.

The next stage of the activity is to introduce the question, “How can you make your picture move?” The first consideration for the students is that the effect of adding a constant to (or subtracting a constant from) all the numbers in the list will alter the image in some way. The teacher is able to test and verify a student’s conjectures using the Viewscreen, observed by the whole class. The whole list is manipulated by using its label. For example, L1 + 1 → L1 would increase all of the X coordinates by one and translate the image one unit to the right.

To achieve a succession of translations, the students need to write a short program with a loop to repeat an instruction. Students are able to build several animations to produce a short film. Samantha’s animation showed the Titanic sailing towards the iceberg, hitting it and sinking.
2.5 General purpose software:

We do rather take it for granted now that any PC, laptop or network station will give us access to a good range of general purpose software – and by default this often consists of elements of the Microsoft *MS Office* suite of software. This contains tools such as a word-processor, spreadsheet, database, slide-shower, web-browser and e-mailer. Many teachers make use of such software for their own preparation of materials outside the classroom. For teaching it is quite possible to prepare an *MS Powerpoint* slide-show, or to use the web-browser to access pages on-line via the Internet, or locally via an Intranet. The one general purpose piece of software which gets most use in mathematics teaching, though, is the spreadsheet. This has its fair share of both adherents and critics – many of whom are quite vocal! So we have taken care to debate the position of spreadsheets within mathematics teaching amongst ourselves and hope that the result gives a useful and balanced view!

**Spreadsheets**

As the name suggests, these were originally developed as accountancy tools and are very widely used in all aspects of finance as well as many other situations involving mathematical and/or statistical modelling. Of course there is no agreed meaningful definition of just what comprises a spreadsheet. But a key feature is that parts of the sheet (usually called `cells`) can be defined in terms of previous ones in such a way that any change to any cell automatically causes the values in every dependent cell to change instantly.

For mathematics we frequently want to be able to set up tables of data, which may use formulae to generate other data, and from which we can draw graphs. For example the figure below shows a table of the minimum braking distances in feet corresponding to speeds in mph, taken from the `Highway Code`. The entries “Speed” and “Distance” in cells A3 and B3 are just labels for the columns. The value 20 has been entered into cell A4. The formula “=A4+10” has been entered into cell A5. This has been `replicated down` from cell A5 to A9. This is an example of the so-called `battleships` notation where each cell has a column letter and a row number. The data for distances has been entered into cells B4:B9.
With the range of cells A3:B9 highlighted, a graph (or chart) can be drawn. In this case it is a scattergram with the data-points joined by line-segments. The range of available graph types usually also reflects the major usage of spreadsheet graphics – such as the display of sales results! Hence it is usually much easier to draw a pie-chart than a box-plot.

In order to try to fit a function to the data we can define some function of the speed from a formula. Here we might try a quadratic function of the form \( d = ks^2 \). So the parameter \( k \) has been entered into cell C4. Now we can define a `model' in column D by defining the cell D4 by the formula: “=C$4*A4^2” and replicating it to the block D4:D9. Note that here we have had to use a mixture of `relative' and `absolute' references so that D5 refers to A5, D6 refers to A6 etc., but that all the formulae use C4. The MS Excel spreadsheet allows you to define names for cells, and so you can use `Insert Name’ to define `s' as the name for the block A4:A9 and `k' as the name for the cell C4. Then the formula for cell D4 can be entered as “=k*s^2” which looks rather more like algebraic notation. Since the name `k' refers to a cell, while `s' refers to a range, we will not have to worry about absolute and relative references when replicating the formula down column D.

Now we can just change the value of `k' in the cell D4 to see the effect on the graph. This gives a powerful way of exploring problems involving data-handling and/or algebra – but some forethought is needed into the best way of laying out the sheet’s design.

So in what ways might you use a spreadsheet in teaching mathematics? You might start with a blank sheet and develop an activity `from scratch’. An example of this is shown in the Y8 case study on the TTA CD-ROM where the class enter their wrist and neck sizes as the data. They can sort the data, for example by wrist size, to find the median and quartiles. They also plot the neck against wrist size as a scatterplot and start to look for a linear model which gives a good `fit’ to the data.

Another approach is to have a pre-designed sheet, known as a `template’, into which students enter their data without having to worry about issues such as formatting. This allows them to spend more time concentrating on the mathematics rather than on the mechanics of the spreadsheet.

Another approach is to obtain ready-to-use sheets designed for a specific task. Examples of these are the materials produced from the Royal Statistical Society’s Centre for Statistical Education such as for census data: http://www.censusatschool.ntu.ac.uk/SpreadGraph.asp and materials available from DISCUS: Discovering Important Statistical Concepts Using Spreadsheets: http://www.mis.coventry.ac.uk/research/discus/discus_home.html.

Below are some further ideas for ways of using spreadsheets, based on example in MS Excel, provided by one of the contributors:

Charts and graphs are dynamically linked to their source data values so that graphical representations of sheet values are automatically re-drawn as source data values change. The appearance (format) of a sheet can be made to vary in response to function values entering “trigger” intervals (`conditional formatting’). On-screen buttons can be defined which make the keyboard unnecessary for entering / changing cell values (an Excel feature on the Forms toolbar) – these are particularly useful on an interactive whiteboard.
There are a number of valuable functions or operations, beyond basic calculation, which offer pupils, at both KS 3 and KS 4, access to interesting ideas. Maximum and minimum values (and other statistical calculations) can be automatically obtained. The Modulo function can be of great value when investigating factors, multiples, or remainders. The Integer function, which returns only the integer part of a value and discards the decimal or fraction component, has many uses including a technique to isolate the individual digits at each place value within a number.

Producing a table for a function of two independent variables can be a very valuable technique. Many contexts of interest even at KS 3 involve more than one independent variable. The illustration here shows part of a table used to discover Pythagorean Triples, each cell in the table contains the square root of the sum of the squares of values in the first row and first column. (In this case conditional formatting has been set to indicate cells that hold integer values)
Useful aspects of spreadsheets in the teaching and learning of mathematics include:

- A spreadsheet is instantly responsive to changed input values which enables exploration of the effect of variables within a process.
- Replication of formulae enables the effortless generation of a large number of new calculated values across a chosen range of source values.
- The spreadsheet as a content-free modelling space - used in this way the spreadsheet creates a model, some dynamic representation of the context under investigation, so that connections between the elements within that context can be considered.
- As an interactive presentation environment.
- On screen buttons enable incremental changes to cell values without using the keyboard.
- Linked values are a defining feature of the spreadsheet. This capability to link and respond enables the spreadsheet to offer interactivity.
- Value sensitive formatting allows the appearance of text, values, and background, to change according to the intentions of the designer, and so give high value feedback, or other information, to the user.

The very wide range of contexts that can be presented in this way allows individual pupils, groups or even the whole class, to explore mathematical relationships with the minimum of materials, where this is an obstacle to learning, and without the repetitive calculations which were previously necessary but added no value for the learner. It is this capability that makes the spreadsheet especially valuable in the production of quality resources for the interactive whiteboard.

**Databases**

There are a number of packages designed to aid sorting and classifying data in a variety of ways, as well as their representation in graphs and charts. These are not specific to mathematics and can be used in any subject where classification according to different criteria is important. They differ from spreadsheets as they can be used to classify and represent non-numeric data. They can be used to support the `PCAI’ data handling cycle. Some packages, such as `Pinpoint’, start from questionnaire design, then automatically sort and classify responses. This shifts the focus from collecting and classifying data to deciding on appropriate representations, interpreting data and evaluating findings. These are the important skills of handling data, often not given sufficient attention. The use by other subject departments of databases, particularly where they prompt discussions about the appropriate forms of representing different data sets, should be part of each school’s agenda for Numeracy Across the Curriculum. Databases linked to surveys can successfully form the focus of group work in mathematics lessons. Using a database for questionnaires and surveys in KS3 will give pupils opportunities to consider appropriate ways of representing data and will equip them for coping with statistical elements of the current GCSE requirements for coursework.

**Wordprocessors**

*MS Office* includes a tool called `Equation Editor` for type-setting mathematical notation. You use the Insert Object menu to open its editor. *MathType 5* is the professional version of the `Equation Editor`. It claims to include new technology for converting *Word* documents to web pages that use high-quality mathematical notation.
2.6 Mathematics teaching software:

Graph plotters

These are probably the most frequently used tools in mathematics teaching. There are several software packages whose primary function is to plot graphs. These include Autograph, Coypu, Omnigraph and Winplot. The built-in software in graphical calculators includes a graph-plotter. Other packages also include facilities to plot graphs. These include dynamic geometry such as Geometer’s Sketchpad 4, computer algebra systems such as Derive and integrated mathematics software such as TI Interactive!

Common features include the ability to:
- use a wide variety of mathematical functions (such as \( \sin(x) \), \( e^x \) etc.);
- show several graphs on the same axes;
- vary the colour, style and weight of graphs of functions;
- label graphs, axes, points etc.;
- rescale the axes easily, including zooming in and out;
- control the way axes and grids are displayed;
- mark points of interest, such as maxima or intersections;
- enter data and display statistical plots.

Different packages offer additional facilities, often with a certain age-range or syllabus in mind. These include tools for calculus, numerical mathematics, statistical distributions, regression and transformation geometry. Most plotters allow function definitions in standard \( y = f(x) \), parametric \( x = f(t) \), \( y = g(t) \) and polar \( r = f(\theta) \) formats. Some plotters allow alternative formats such as \( x = f(y) \) and implicit definitions: \( f(x,y) = k \). Some also have tools for the exploration and solution of differential equations.

Below are example of plots of \( y = 1/(x-1) \) and \( y = 1/x \) from Autograph and Coypu.

If you plan to use graph-plotters for whole class displays then it will be important to be able to select line styles and colours which are very visible. Of course graph-plotters are also very useful outside the classroom so that you can copy and paste graphs into documents such as task-sheets and examination papers!
An activity that has validity at almost any level is to plot the graph of function whose equation is hidden from view and to try to find an equation whose graph matches it. (If you use this activity with graphical calculators it is useful to have graphs pre-printed on pieces of acetate, such as OHTs, which can be overlaid on the screen.) Below are some examples extracted from pp 85-6 of ‘Teaching Mathematics with ICT’.

Dynamic geometry software

This software has evolved from a package written for the Apple II computer over 20 years ago called the Geometric Supposer, written by Judah Schwarz and others at MIT. Essentially it provides a range of tools for you to construct geometric objects from a range of ‘primitive’ objects (points, segments, lines, circles etc.) using both “classical” constructions (midpoint, perpendicular, parallel etc.) and transformations (reflect, rotate, translate etc.) Once drawn, measurements can be taken from objects (length, angle, area etc.). The ‘dynamic’ aspect comes from the ability to drag defining objects, such as points, round the screen to deform the resulting shape – and hence to be able to look for aspects that always stay the same (‘invariants’). The dragging is usually done with some analog control device. Originally this was the ‘game paddle’ of the Apple II. Now it is usually the mouse, or tracker-ball, of the PC or laptop. However dynamic geometry software (DGS) is ideally suited for use with an interactive whiteboard where the dragging can be done with a stylus or even the finger. The two most common DGS packages for schools are Cabri Géomètre and the Geometer’s Sketchpad (GSP). There is also a free DGS package called Wingeom in the Peanuts software series obtainable from Richard Parris at: http://math.exeter.edu/rparris . Another package called Cinderella has been particularly designed with the production of interactive web-pages in mind. (Most DGS packages now have facilities to export ‘live’ diagrams in Javascript.) MathsNet maintains a page detailing these packages at http://www.mathsnet.net/dynamic/ and also has many examples of interactive geometry web-pages.
As an example we will show the construction for the circumcentre $O$ (and hence the circumcircle) of a triangle $ABC$ using Sketchpad. The points $A,B,C$ have been constructed and segments drawn between them. With segment $AB$ highlighted the Midpoint tool is selected from the Construction menu. With both the midpoint and the segment $AB$ selected the Perpendicular tool is selected to construct the perpendicular bisector of $AB$. A similar sequence is used to construct the perpendicular bisector of $AC$. With both perpendicular bisectors highlighted $O$ is constructed with the Intersection tool from the Construction menu. The display of the figure can be adjusted by selecting differing line styles, weights and colours for segments and lines, and by labelling some or all of the points in a variety of fonts, sizes and styles. Using the mouse you can drag any of the points $A$, $B$ or $C$ around independently to deform the triangle $ABC$. This shows some of the similarities and differences between drawing by hand and with DGS software.

With the dynamic image we can test hypotheses such as: if we drag $A$ around

- does $O$ always lie within the triangle;
- for what sort of triangle does $O$ lie below $BC$;
- when will $O$ lie on $BC$;
- what is the locus of $O$…?

Here we start to arrive at a real issue in the use of DGS – the software can’t prove anything! It can give us lots of clues, but we have to be alert to the geometric reasoning that is needed to establish results like $O$ is the point equidistant from $A, B$ and $C$. In the second of the pair of diagrams above we have highlighted the triangle $AOB$. If we can assume that we already know that the locus of points equidistant from $A$ and $B$ is the perpendicular bisector of $AB$, then we can deduce that $OA = OB$ and hence that $AOB$ is isosceles. Similarly we can deduce that $BOC$ is isosceles, and hence that $OB = OA = OC$. Which explains why $BOC$ must also be isosceles and why $O$ must lie on the perpendicular bisector of $BC$. 

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The report ‘Teaching and Learning Geometry 11-19’ by the Royal Society/JMC working group recommends the increased use of ICT in teaching geometry and says:

Geometry teaching outside primary schools can be, and has been, conducted with a minimal amount of equipment - such as a stick of chalk and a piece of string (echoing images of ancient Greeks drawing in the sand). Our view is that teachers should now have at their disposal an appropriate variety of equipment from which to select, depending on fitness for purpose. In particular we wish to see the potential of ICT realised in supporting the teaching and learning of geometry. There is already software available, such as for Dynamic Geometry, but its use is not widespread. Many schools do not have licences for the software. There is also a need for the development of additional software, such as to support work in 3 dimensions. Increasing numbers of schools and colleges are now being equipped with ‘interactive whiteboards’ - where a computer image is projected onto a touch sensitive screen. This medium has considerable potential for interactive whole-class teaching of geometry. We would like to see the funding to schools for ICT being used more effectively to support the geometry curriculum.

The ATM publish a useful set of resources for teaching geometry using Cabri-Géomètre and The Geometer's Sketchpad called Active Geometry. These comprise 17 files and over 100 associated worksheets offering various activities on exploring mainstream topics in shape and space.

As well as tools for constructions, transformations and measurement, DGS usually include the means of drawing loci, performing animations and working with coordinates. Both Cabri and Sketchpad include a Calculator tool which enables measurements to be combined. Measurements, and functions of them, can thus be used to define the coordinates of a point plotted in an axis system – and the locus of such a point will represent the graph of function. Hence DGS are not restricted solely to geometric applications but can bridge neatly into algebra. As DGS evolves it will embrace more tools already associated with other forms of mathematical software. For example a new feature in version 4 of Sketchpad is the inclusion of graph-plotting tools. The illustration below shows a modelling situation where a segment $XY$ has been set up to represent the length of a piece of string which is used to enclose a rectangle $ABCD$, whose area and side have been measured. The point $A$ can slide on the segment formed by $X$ and the midpoint $D$ of $XY$. Thus $A$ is the independent variable with segment $XD$ as its domain. The values of the distance $AD$ and area $ABCD$ are used to define the coordinates of the point $P$ plotted on the axis system. As $A$ slides on $XD$ so $P$ traces out the graph of area against side – which is shown as the locus of the dependent variable $P$ against $A$. The analytic form of this curve can be derived, entered as a function $f(x)$, and graphed to compare with the locus.
Data-handling tools

A wide range of ICT tools has been developed for handling data. These tools facilitate the collection, communication, analysis and interpretation of data. Tools include data loggers, graphical calculators, the Internet and specific software for computers. ICT tools enable students to:

- formulate questions;
- undertake purposeful enquiries;
- collect and represent data;
- engage in practical and experimental work;
- appreciate randomness;
- question the representation of data;
- draw and criticise conclusions.

ICT tools provide the opportunity to work with data that has meaning to students so that they have a measure of ownership that is real to them. They aid students in specifying problems, in testing hypotheses, in dispelling myths, and collecting data. Large data sets can easily be gathered, e.g. from the Internet or the KS3 Mathematics Strategy CD-ROM. They can be differentially sectioned within the software. The resulting data sets can then be explored by students using whatever is the most convenient ICT tool. For example they can be distributed to graphical calculators for analysis individually by students. This exploratory analysis is supplemented through built in analytical procedures and tests which can lead to interpretation and discussion, before returning results to the PC in order to write up a report.
Fundamental concepts of statistics are made accessible through the use of ICT. For example, the nature of data and the distinction between discrete and continuous data can be brought to life using a radiation monitor to collect times of random events. The resulting data could then be modelled by a Poisson distribution or an exponential distribution. The distinction between the data and the scale used to record the data becomes a reality for students.

Bivariate data, and the concepts of correlation and regression, can be easily explored through the use of data logging equipment. The use of one distribution to approximate probabilities for another can be explored graphically and numerically. ICT provides many opportunities for students to consider issues surrounding sampling, such as methods, stratification, repeated measures, random samples, convenience samples, sample sizes and bias.

Many of the ICT tools listed separately here can be readily employed for data-handling work, such as spreadsheets, graphical calculator, data-loggers and integrated mathematics packages. There are also special purpose tools designed for educational data-handling. Perhaps the most interesting of these is the ‘Fathom’ package from Key Curriculum Press which can be obtained from both Chartwell-Yorke and QED. Another shareware package from Peanut software http://math.exeter.edu/rparris/ is WinStat. The figure below shows the fitting of a linear regression model to wrist and neck measurements using Fathom.

![Fathom](image)

**Computer algebra systems**

Originally computers were designed for civil and military task involving the manipulation and computation of numeric data. But as computer tools were developed for working with strings of symbols it became apparent that many of the rules we use for symbol manipulation in algebra and calculus could also be turned into computer algorithms – an obvious example is the rule for differentiating \( x^n \) where \( x \) is real and \( n \) is an integer. Such tools were developed for mainframe computers to handle the vast amount of routine manipulation...
involved in applications such as in theoretical physics – where their complexity meant any attempt to carry these out by hand would lead to many errors. From these a variety of packages were developed to help researchers in mathematics and physics, onto which a variety of facilities were added to make them useful for teaching and learning as well. Examples of these Computer Algebra System (CAS) packages include Mathematica, Maple and Reduce. Of considerable educational importance was the development of the Derive package with many of the features of its predecessors, but designed to run on common PCs at a price education could afford – it used to be advertised as “a college education for $200”. Another educational version of CAS is called LiveMath. CAS is also available on a number of hand-held platforms, including some graphical calculators. As with ordinary calculators, which throw into question just how much arithmetic facility we now need, so CAS raises similar issues with respect to symbolic facility in algebra and calculus. The RS/JMC report ‘Teaching and Learning Algebra pre-19’ raises some of these issues. The Mathematical Association’s report ‘Symbolic Manipulation by Computers and Calculators’ addresses these further. The ATM’s ‘Computer Algebra at A-level’ focuses more closely on issues in 16-19 mathematics.

One problem with symbol manipulation is that there can be many equivalent forms of output and the software usually has to “guess” which form the user expects. But in teaching this may be something which can be exploited to advantage. As an example the figure shows some manipulation to solve an equation and to check the result.

While most examination syllabuses currently prohibit the use of CAS there are exceptions. Both the Scottish Highers and the International Baccalaureate are planning to introduce their use in the near future. Whether or not students are allowed to use CAS in examinations they can still be of benefit both to teachers and to learners. Since they contain many features of a more advanced nature, such as for calculus, complex numbers, matrices etc their main use may extend beyond the KS3/4 curriculum.

**Integrated mathematics packages**

We have already seen how different types of mathematical software can have very different roots – for example DGS was explicitly developed for an educational context while CAS was been developed mainly for academic research. Many working engineers and applied mathematicians needed rather different tools which would allow them to set up problems for numerical solution, produce results, test them and write a report explaining the process. Mathsoft’s MathCad was, and is, the major package for such ‘computer-assisted mathematical modelling’. As it has been through many refinements earlier versions of the software have been made available at educational prices. The current educational version is called Studyworks Maths Deluxe 2002. A review by Neil Challis appears on the LTSN site: [http://ltsn.mathstore.ac.uk/newsletter/feb2002/pdf/studyworks.pdf](http://ltsn.mathstore.ac.uk/newsletter/feb2002/pdf/studyworks.pdf).

By contrast Texas Instrument’s TI Interactive! was developed explicitly for education and contains a blend of functions found in (a) MathCad, (b) Derive, (c) the TI-83 graphical calculator and (d) MS Office! It was this range of functions that led to TI Interactive! (TII) being adopted as the main mathematics tool, along with graphical calculators and the Geometer’s Sketchpad, for the DfES Year 7 pilot project: MathsAlive! Its features include:
• a graph-plotter for functions and data;
• a spreadsheet – a subset of MS Excel;
• a symbol manipulator – very similar to Derive;
• a data editor with facilities for data-handling, probability and statistics with lists;
• a word-processor – a subset of MS Word;
• a web-browser with easy means of capturing data into lists – a subset of MS Internet Explorer;
• an e-mailer – a subset of MS Outlook Express;
• live mathematics boxes where values and functions may be defined and altered so that any dependent boxes, graphs etc. are recomputed when changes occur;
• an easy interface with data-loggers (CBR, CBL) and graphical calculators (TI-83) for capture and exchange of data;
• easy ways to transporting data and documents using a variety of compatible file formats.

You can click on any live part of a TII document to edit it – and used with large fonts, thick lines etc. it is an ideal medium for whole class displays of mathematics such as with an interactive whiteboard. Put simply, the view of the group which produced this advice document is that “it is the best thing since sliced bread!” – especially since a single copy costs around £30 and a licence about £10 per station.

A rectangle has a perimeter of 10 and a side length of $x$ units.
So its breadth is given by 5-$x$ units and its area by $x(5-x)$ square units.
We can explore both the table and the graph of this function:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x(5-x)$</th>
<th>$x$</th>
<th>$x(5-x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>2.1</td>
<td>6.09</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>2.2</td>
<td>6.16</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>2.3</td>
<td>6.21</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>2.4</td>
<td>6.24</td>
</tr>
</tbody>
</table>

And if we knew some calculus we could do some manipulation!

$$\frac{d}{dx} (x(5-x)) \quad \text{solve} \{ans = 0, x\} \quad x(5-x) \bigg|_{ans}$$

$$5 - 2x$$

$$x = \frac{5}{2} \quad \frac{25}{4}$$
A new, and rather different, integrated package with a very different interface is *Crocodile Mathematisc*. Described as a user-friendly, interactive mathematics laboratory it allows both teachers and pupils to create and experiment with mathematical models by linking shapes, equations, numbers, graphs, text and pictures. It allows users to set up mathematical models by dragging and dropping shapes and tools onto the screen. The software was launched at the BETT exhibition on January 9 this year.

2.7 Integrated Learning Systems

An Integrated Learning System (ILS) consists of curriculum software with a built-in management system. This means that the software records all the work that each individual student does. Student reports can be produced when required. These reports can then be used to identify individuals’ strengths and weaknesses. Two of the most common ILS systems in UK secondary schools are `Successmaker` and `Headstart`. `Successmaker` includes two main mathematics courses – `Concepts and Skills` and `Investigations`. `Concepts and Skills` can give students practice in basic work. The `Investigations` course is a suite of modules providing challenges and open work – probably for more able students. However you will need to be selective with these modules. `Successmaker` is not only for mathematics but includes English (and science) curriculum software as well. `Headstart` began as an ILS to support GCSE Mathematics. The original product claimed to cover the National Curriculum programmes of study at Key Stage 4 for level 6 and above. However it has recently added a Foundation version covering Levels 2-5 of the National Curriculum.

Schools need to have a clear rationale for using this type of software - aims and objectives need to have a clear focus. The implementation model is critical to the success of the system and needs to be monitored and perhaps adapted over time. Some schools encourage their students to use ILS at breakfast clubs, lunchtime, or sometimes after school. Use during lesson time poses a number of management issues. Some schools withdraw students from normal lessons to use ILS. However this has the disadvantage of students missing some of their timetabled lessons. The model of use needs to be considered in the light of the current National Strategies. The cost of ILS is quite considerable and schools will need to consider carefully whether better value for money could be achieved by using other ICT approaches.

Teachers need appropriate training on the use of these systems. This is normally provided by the supplier as part of the purchase price. Students should have an induction into the courses and not just be left to start alone. The courses should have appropriate regular use by particular students. For example, daily use is recommended for the basic `Concepts and Skills` course of `Successmaker`. It seems to work better if the students can have some form of supervision. It is recommended that reports should be printed off regularly to inform staff of student progress. These should then be analysed and used to help inform future teaching and learning.

Independent evaluation and advice reports can be obtained from BECTa. The suppliers of the software should be able to give information about successful implementation sites. Information about `Successmaker` can be obtained from RM Learning Systems, New Mill House, 183, Milton Park, Abingdon, Oxon, OX14 4SE. Tel. 0870 908 6700. E-mail address: rmls@rm.com. Information about `Headstart` can be obtained from Headstart Software Marketing Ltd, Vauxhall Industrial Estate, Ruabon, Wrexham, LL14 6AH. Tel. 01978 810000. E-mail address: post@headstartsoftware.co.uk.
2.8 Data-loggers

Perhaps it is surprising to see these included when their more obvious curriculum "homes" would seem to be science, D&T and ICT. Of course the forging of cross-curricular links using ICT as a catalyst cannot be a bad thing. But there are many aspects of the mathematics curriculum whose origins lie in modelling scientific phenomena and which can be brought to life through practical contexts. The most obvious of these is the interpretation of distance-time graphs which can be greatly facilitated with the use of a motion detector (i.e. a distance measurer). Distance data may include someone walking to and fro, or a pendulum swinging, or a model car running down a ramp, or a ball bouncing, or a spring-mass system in oscillation etc. Once the data have been captured using a data-logger then they can be manipulated, displayed, analysed etc. using any of the normal ICT tools for data-handling. Thus an experiment can be performed in front of the whole class, and the data then distributed for group, paired or individual work.

There is a wide range of data-capture systems and accompanying probes in use in other subjects, but we will concentrate on two particular devices designed for use both with graphical calculators and PCs. These are the Texas Instruments Calculator Based Ranger (CBR) and Calculator Based Laboratory (CBL). The first of these is a device about the size of a pack of three golf-balls which emits and collects an ultra-sound signal (like the way a bat navigates in flight). The second is a more versatile control unit which can be connected to a number of probes simultaneously – such as for temperature, light intensity and voltage. The Key Stage 3 Framework for mathematics includes examples of the use of both devices, e.g. for the successive bounces of a ball, and for the cooling of a liquid. They can be used throughout the 11-19 curriculum providing data for modelling using linear, quadratic, inverse, sine and exponential functions.

An account of their use by mathematics and science teachers at KS3 can be found in the BECTa publication 'Data-capture and modelling in mathematics and science'. This can be downloaded from: http://vtc.ngfl.gov.uk/uploads/application/datacapture-16796.pdf

In 1988/9 the TTA provided a teacher-researcher grant to accompany a project in Hampshire to explore the use of hand-held ICT (TI-83/CBR/CBL) across the curriculum in secondary schools - involving mathematics, science, D&T, geography, PE, PSHE etc. The 4-page abstract report from Peter Ransom, Mountbatten School, Romsey can be downloaded from: http://www.canteach.gov.uk/research/grant/98-99/Peter_Ransom.pdf

The TI-83 Plus graphical calculator, as used in RM Maths Alive, has a built-in application ‘CBL/CBR’ to facilitate the use of data-loggers. In this example we capture distances walked in a straight line.

You can use the left and right cursor keys to trace and read off from the graph. You can also make "qualitative" observations, such as "walking away", "standing still", walking towards", "constant speed", "faster", "slower", "longer", "shorter" etc. You can take measurements from the graph and thus can make "quantitative" remarks such as "in the first 4.4 seconds you moved from 0.46m to 2.09m away" or "your average speed was 0.37 m/s". A popular
activity is to draw a target distance-time graph on the whiteboard, or OHT overlay, and to ask students to try to match it. When you choose to "QUIT" the application, the time data (in seconds) are stored in list L1 and displacement data (in metres) are in list L2. Pressing [GRAPH] redisplay the last graph shown, and by pressing [TRACE] you can take readings from it. So you can now try to "fit" a model to any part of the motion. Enter a function in Y1 which you think might match the first part of the distance-time graph.

The screens above show a first attempt - can you improve on it?
Now think hard about the various terms in the definition of Y1 above. Y1(X) is a measurement in metres of the distance from the CBR at any time X in seconds. So when you use a function like Y1(X) = 0.37X+0.46 it is clear that 0.46 must also be measured in metres, and that Y1(0)=0.46, so that it is the distance away from the CBR when the time is 0, i.e. the starting, or "initial", displacement. Similarly 0.37X must be in metres, but X is in seconds, so 0.37 must be in metres/sec (or ms^{-1}) and so is a speed (or, since it can be negative, strictly a "velocity"). Hence the gradient of the line represents the average, or constant, velocity.

An alternative to hand-held technology is given by Tim Morland:

_A lot of emphasis has been given to the place of 'handheld technology' in school Mathematics, and for good reasons. We have chosen another route, largely because we have reasonable access for our students to desktop PCs and also because it seems unnecessary to duplicate resources when the school's science department are equipped with data-logging equipment. To show the sort of thing that we have done, I would refer enquirers to my article “Modelling a Simple Mechanical System” in the IMA journal “Teaching Mathematics and its Applications” (OUP), Volume 18, No.2._

The necessary mathematical background required for students is

- Hooke’s Law
- Newton’s Laws
- Solution of linear differential equations

This would be covered by most students preparing for a Further Mathematics qualification.

One experiment uses the physical apparatus shown opposite, and the displacement of the lower body is recorded after the system is set in motion according to a variety of initial conditions.

The actual motion of the body is compared to the predictions of a mathematical model constructed by students. After some refinement of the model, satisfying agreement can be reached.
2.9 Sources of data and mathematical stimulus: the Internet and CD-ROMS

The Internet

There are some technological changes which revolutionise education. It is difficult to imagine how we coped before the advent of the photo-copier! The same is true of the Internet. It is an unparalleled mechanism for making available to anyone, almost instantly, and at near-zero cost, anything that is expressible on a PC. Fortunately there is already a good deal of both advice and resources to support ICT use in mathematics already there waiting to be found! In the next section we give a selective list, such as those available from BECTa and the Virtual Teacher Centre (VTC). One of the most significant sites for UK mathematics is MathsNet www.mathsnet.net maintained by a teacher: Bryan Dye and supported by Granada Learning. Everyone will have their own personal favourites, and web-sites come and go surprisingly quickly, but we hope the list below forms a good starting point.

- MA - The Mathematical Association http://www.m-a.org.uk
- ATM - The Association of Teachers of Mathematics http://www.atm.org.uk
- Count On – the follow up from MathsYear 2000 http://www.counton.org
- NRICH http://www.nrich.maths.org
- VTC the Virtual Teachers Centre - Choose teaching subjects and then mathematics http://vtc.ngfl.gov.uk
- NC Online - Choose mathematics http://www.nc.uk.net
- Sidney Tyrrell’s personal site for Statistics http://www.mis.coventry.ac.uk/~styrrell/resource.htm
- MathsNet http://www.mathsnet.net
- Douglas Butler at Oundle School has a site full of valuable links http://www.argonet.co.uk/oundlesch/mlink.html
- TI graphical calculator support http://education.ti.com/uk/teacher/teacher.html
- Ambleside Numeracy http://www.ambleside.schoolzone.co.uk/ambleweb/numeracy.htm
- BBC http://www.bbc.co.uk/education/
- MathAid http://www.mathaid.com
- Explore Math http://www.exploremath.com/
- World of Mathematics http://mathworld.wolfram.com/
- Maths Online http://www.univie.ac.at/future.media/moe/
Our general advice to you is to begin with a fairly clear sense of what learning experience the resources are to support, before casting about for helpful resource-providing websites. But you also need to allow yourself some undirected browsing to stimulate your imagination and to widen your awareness of new possibilities.

As with reviewing software, some criteria are needed when reviewing mathematics websites. We have used the following criteria in drawing up our recommendations:

• ease of navigation around the site;
• transparency of design;
• emphasis of content over design;
• a large body of interactive material available;
• high relevance of content to school use.

The emphasis above is on “interactive”: meaning pages that require action by the user, often through varied and continual use of the mouse. This may involve Macromedia Flash and Shockwave, Java or JavaScript, though these technicalities should not be apparent to the user - apart from issues of download time. Some sites dress material up with “attractive” graphics, or provide sophisticated but cumbersome methods of getting at the material. Unfortunately some of the BBC’s material (BiteSize) suffers from this.

There are sites catering for specific areas of Mathematics at Advanced Level, particularly Statistics. Four excellent examples are:

• Inferential Stats at http://faculty.vassar.edu/~lowry/webtext.html
• Rice Virtual Lab in Statistics at http://www.ruf.rice.edu/~lane/rvls.html
• CAST at http://cast.massey.ac.nz/
• Calc101 at http://www.calc101.com

Other sites offer excellent research and background material, such as:

• the History of Maths site at St Andrews http://www-groups.dcs.st-and.ac.uk/~history/
• Ron Knott’s Fibonacci numbers site at http://www.ee.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html
• The Interactive Mathematics Miscellany and Puzzles site at http://www.cut-the-knot.org/

There are also excellent online strategy puzzles scattered about the web, such as chess, go, reversi, draughts, Chinese checkers, though no one site has a comprehensive selection. MathsNet has made a small collection of them at www.mathsnet.net/puzzles.html.

MathsNet includes a huge collection of interactive material for use in schools using online versions of dynamic geometry packages at all Key Stages at www.mathsnet.net/geometry/. The site covers Shape, Transformations and Geometric construction in detail, along with smaller topics, such as Euclid’s elements, construction without a ruler, properties of triangles and circles.

Jennifer Piggott, NRICH Project Director, brings a slightly different perspective to the question “what use is the Internet?”

In order to make effective use of an online resource it is essential to know what the site is trying to achieve and why it will suit your purpose.
These resources can take a number of forms from lesson plans to subject specific software through to pure information sources, such as sites containing population statistics. The first requirement is for you to be clear about the type of resource you are looking for so that you can begin to look in the right places. Sites will offer support in a variety of forms and it is useful to be aware of what a site is trying to achieve so that you know what to expect. The rest of this section looks at types of sites and why you might find them useful.

The range of places to start searching for mathematics resources is growing. Sites like the BBC [http://bbc.co.uk/learning](http://bbc.co.uk/learning) offer opportunities for revision and some, such as the NGfL [http://ngfl.gov.uk](http://ngfl.gov.uk) and Curriculum Online [http://www.nc.uk.net/home.html](http://www.nc.uk.net/home.html), simply refer to further sites and resources (rather like a compendium). There are also sites that have specific, curriculum linked, resources such as Counton [http://www.counton.org](http://www.counton.org) and the NRICH online mathematics club [http://nrich.maths.org](http://nrich.maths.org), which offers enrichment materials. This site contains a wealth of problems, games and articles and offers a search facility for locating material related to particular aspects of mathematics. There is also a mathematical thesaurus, which is being continually updated, and gives access to definitions and explanations of mathematical term and makes links to related terms. Access to online discussion groups, like the ones found on the NRICH site enable pupils, teachers and parents to discuss mathematics with peers. Finally, simply searching using a search engine such as Google can also be useful.

One problem is the vast number of sites and finding time to assess the potential of each one. The NGfL “flags” sites they feel meet their assessment criteria and TEEM [http://www.TEEM.org.uk](http://www.TEEM.org.uk) is beginning to evaluate web based resources as well as multimedia software. These evaluations are carried out by teachers who use the resource(s) with their pupils over several lessons and therefore they have the advantage of being evaluated in the “real world”.

This is all beginning to show how complicated the task of identifying and using resources can be. The answer is to start small. Get to know one resource well and share it with colleagues. The site you pick will reflect your approach to teaching mathematics so your choice may be quite different from someone else’s, simply because you are a different teacher. As your confidence grows you can move on or use a network of colleagues to develop supporting materials for pupils to use with online resources. Many hands make light work.

Ron Knott adds these comments:

The advantages of the Internet over other software are that:

- browsers are free
- pages are available at home as well as school and everywhere else
- students’ work and teachers’ ideas can be shared world-wide
- platform independent
- useful for student revision later
- students who missed class can easily catch up
- keen students can carry on working on their own using the resource
- web-generated interactive test pages give instant feedback to students and allow plenty of practice.

However, ease of use of the internet is predicated on interactive whiteboards and each classroom having an online connection and monitor projector.
CD ROM

CD ROMs provide an alternative storage and delivery mechanism in the same sense as the Internet, but with some significant differences.

- They may be more reliable, and load faster, across a multi-user school network.
- They may make pupils’ learning more manageable by offering scope within boundaries. The structure of a CD-ROM may prevent students following web-trails in all directions, which can quickly lead to their becoming “off-task”.
- They do not have the same quality of currency when compared to Internet-based resources, which may not be highly significant in all cases but needs consideration.

One large scale use of CD-ROMs in mathematics is for item-banking past test and examination questions, often with tools to enable you to construct your own test papers.

2.10 Digital image technology

The increased availability of digital cameras and video camcorders provides a valuable source of geometric and other data which can be used in a number of ways. For example a photograph can be captured from a digital camera, as a still from a video camcorder or scanned from an original or copy. The image can be manipulated in standard imaging software and the result saved in a standard image file format or pasted to the clipboard. The resulting image or file can then be opened in software such as the Geometer’s Sketchpad and constructions, measurements etc. made using the image. The figure below shows three points $A,B,C$ superimposed on the lower curved span of the bridge, and used to fit a circular arc.

\[ m \overline{FA} = 10.89 \text{ cm} \]
\[ m \overline{AC} = 12.20 \text{ cm} \]
Still images and video clips are also available on CD-ROMs and via the Internet. For example the Problem Pictures CD-ROM [http://www.badsey.net/pp/index.htm](http://www.badsey.net/pp/index.htm) produced by Richard Phillips provides a massive base of digital images together with notes suitable for use in teaching and learning aspect such as ratio and proportion, symmetry, 3D visualisation etc.

The Multimedia Motion II CD-ROM [http://www.csmedia.demon.co.uk](http://www.csmedia.demon.co.uk) from Cambridge Science Media provides a large number of video clips of actions such as the striking of a golf ball, the path of a squash ball and the collision of a vehicle together with tools for recording coordinates of objects from successive frames so that distance-time graphs can be drawn, velocities and accelerations estimated etc.

Images from a digital camera (or scanned images from photographs, books etc) can be imported into a variety of graphic software, such as MS Paint, from which cursor positions can be read off as co-ordinates and used as data sets for modelling. Frames from a digital video camcorder (or stored input from a VCR or TV) can be similarly digitised manually with the frame number and film speed giving the time data. For examples of each of these see Teaching Mathematics with ICT, Oldknow & Taylor, Continuum 2000.

2.11 Video conferencing

This consists of much more than just making a telephone call with pictures!

Features of the technology include the following.

- At any location, the image and sound can be projected to be available to a group rather than just an individual.
- The connected activity can involve more than two parties/locations simultaneously.
- The technology supports shared sight and control of applications, simulating the experience of working together at the same machine.

In order for it to be an effective means of communication, participants need to be able to:

- see the other people person to feel connected;
- see the objects of discussion;
- have shared control/manipulation of the objects of discussion.

Three of the major models of use for mathematics work are:

- Conference – this is ‘presenter centred’ (such as in the Motivate activities of the NRICH Project);
- Workshop – which may be a significant model for teachers’ professional development as well as for activities with students;
- Peer to peer – where teachers and/or pupils work in a style of collaboration closer to the familiar work/learning style in school.

A successful case study of video-conferencing in mathematics contained the following elements:
• 12 students prepared in advance for a conference using activities located on the web;
• they introduced themselves and shared findings with other schools in the conference;
• the presenter picked up points and provided an input, after which the students
  presented results to others during the conference;
• a final task was set for pairs of schools to work together on an aspect of the topic of
  the conference;
• notes and support were provided on the web for teachers and students;
• in a second session schools reported back their findings and reviewed the work –
  which led to extensions and generalisations.

2.12 Courseware

A number of published schemes, such as the *Folens Maths Programme* and *Key Maths*, are
now including materials making use of ICT such as graphical calculators and spreadsheets.
Similarly there are supporting materials to accompany A/AS schemes such as MEI and
SMP16-19. *Absorb Mathematics* software from *Crocodile-Clips* is their new courseware
for GCSE Mathematics. It combines Crocodile Mathematics models with animations and

The current moves towards a ‘digital curriculum’ mean that organisations, principally the
BBC, will provide large amounts of downloadable materials to accompany the National
Curriculum and GCSE courses.

At the moment, though, there is a unique ICT based mathematics course in Research
Machine’s *Maths Alive Framework Edition* (MAFE) which has been developed from the
DfES Y7 project to cover the whole of the objectives in the KS3 Framework for
Mathematics. This is based on the use of the mixed model of hardware described in section
1 above, which can include:

- Interactive whiteboard, projector, PC, printer and Internet connection
- Further PCs to support group work
- Teacher’s graphical calculator displayed via video adaptor and/or OHP pad
- Graphical calculators and data-loggers for student’s paired or individual use.

The software can include:

- *RM Easiteach Maths* for use with the interactive whiteboard
- *MS Office* elements; *Word, Powerpoint, Excel, Internet Explorer*..
- Small software, videos and stimulus materials written by RM’s 3T production company
- *MSW Logo*
- *Geometer’s Sketchpad V4*
- *TI Interactive!*

The service includes a large number of supporting files containing materials to support
whole-class work, student activities, mental-and-oral starters, task sheets, homework sheets,
assessment sheets, teachers’ notes, students’ notes etc. A sample of five small programs:
*Car Jump, Code Breaker, Crazy Paving, Loop Cards* and *Matching Pairs* can be found on
the BECTa site: [http://www.becta.org.uk/ks3online/maths/index.html](http://www.becta.org.uk/ks3online/maths/index.html)

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2.13 Overview

We hope that we have convinced you that there is no shortage of ICT resources for teaching mathematics at all levels! While we have tried to give a fair coverage of the range of materials available we are aware that we cannot be exhaustive. A question often asked though is “what should be the absolute baseline for ICT resources in mathematics department?” So, with a certain amount of trepidation, here are some pointers we hope you will find useful.

Hardware:
1. Some means of supporting whole-class work with ICT to include any of the following:
   • PC/laptops displayed with interactive whiteboards, data projectors and/or large screens;
   • Graphical calculators/data-loggers displayed with OHP pad and/or video adaptor.

2. Some means of supporting paired and/or individual access, preferably in a normal classroom including any of the following:
   • networked PCs;
   • laptops;
   • hand-held devices such as graphical calculators, data-loggers and palm-tops.

Software – with site-licences:
3. A version of Logo, such as the free MSW Logo.
4. Dynamic Geometry Software, such as the Geometer's Sketchpad or Cabri.
5. Other mathematics teaching software for graphing, data-handling etc, such as the integrated maths package TI Interactive! or individual titles such as Omnigraph, Coypu, Autograph etc.
6. Software and cables to support the easy transfer of data, programs, applications etc. between PCs, graphical calculators, data-loggers and the Internet.
7. A library of resource materials such as small software from SMILE, stimulus materials on CD-ROM such as Problem Pictures etc.
8. Resources to support the use of general purpose tools such as those in MS Office.

Materials:
9. Publications to support mathematics teachers in their use of ICT, including those from TTA and BECTa, journals such as ATM’s Micromath and books such as Oldknow and Taylor’s ‘Teaching Mathematics with ICT’.

In selecting mathematical ICT tools such as software and graphical calculators bear in mind the ratio of time taken to become familiar with the tool and curriculum opportunities for its use! Tools such as dynamic geometry software, integrated mathematics software, graphical calculators and data-loggers are deployable at all levels from Y7 through to Further Maths.

Maybe now would be a good time to take a look at what the 2000 National Curriculum includes under Breadth of study for Mathematics key stage 4 higher:

During the key stage, pupils should be taught the knowledge, skills and understanding through...choosing appropriate ICT tools and using these to solve numerical and graphical problems, to represent and manipulate geometrical configurations and to present and analyse data.

So, do you think you’ve got that sorted now?
Section 3: Supplementary information

We conclude with a collection of references to supporting materials, information about ICT support for teachers, references to past trials of ICT in mathematics and on-going projects, as well as a glossary of the acronyms which go with computers and government initiatives!

3.1 Current sources of information and materials

The following documents are valuable primary sources of information:

General
BECTa *Mathematics and IT - A pupils entitlement*  

NGfL *Transforming the Way we Learn*  
http://www.dfes.gov.uk/ictfutures

Ofsted *ICT in Schools – Secondary mathematics*  
HM1 705, June 2002  


RM *Good Practice in the Use of ICT*  
http://www.rm.com/reports/

TTA *Identification of Training Needs*  
http://www.canteach.gov.uk/info/ict/nof/paper.htm

The following web-sites contain valuable information and links, and are described below:

Websites
The Virtual Teacher Centre (VTC)  
http://www.vtc.ngfl.gov.uk/

The Teacher Resource Exchange (TRE)  
http://tre.ngfl.gov.uk/

The BECTa Educational Software Database (BESD)  
http://besd.ngfl.gov.uk/

The BECTa ICT Advice Site  
http://www.ictadvice.org.uk/

The BECTa Website  
http://www.becta.org.uk/

The Further Education Resources for Learning (FERL)  
http://ferl.becta.org.uk/

The Virtual Teacher Centre (VTC)
The mathematics area on the VTC is divided into the following sections:

- Subject resources: containing links to web sites that have been approved as having useful curriculum content, as well as the Teacher Resource Exchange and other curriculum resources.
- The curriculum: providing direct links to the National Curriculum documents, schemes of work and frameworks for teaching for mathematics.
- ICT support: here you will find information, advice, case studies and resources for using ICT in mathematics. (See below for more detail)
- Tests and exams: this section provides links to the on-line examination syllabuses for Mathematics and links to revision materials. There is information and support for SATs.
- Organisations: here you will find links to subject associations, national projects and other mathematics related bodies.
- Discussion groups: take the opportunity to communicate, discuss and share ideas with other mathematics teachers.
The ICT support pages for mathematics  

These contain the following sections:

- **ICT and Mathematics**  
  A simple guide to ways in which ICT can support pupils learning mathematics in both primary and secondary phases

- **ICT materials**  
  A catalogue of software and equipment for mathematics together with some suggestions and advice about their use.

- **ICT in Practice**  
  This section contains classroom ideas and activities that focus on integrating ICT with the teaching and learning of mathematics. It contains a large number of resources and ideas for the use of graphical calculators here.

- **Managing ICT**  
  This provides case studies of six schools developing their use of ICT and a departmental audit of ICT use in mathematics

- **Internet resources**  
  Some links to mathematics resources on the internet

The Teacher Resource Exchange (TRE)

The Teacher Resource Exchange is designed to help teachers develop and share ideas for activities and resources.

Contributions are many and varied from simple ideas and questions, to complete contributions/lesson plans or schemes of work, which will enable other teachers to use these resources within their own lessons.

Registering with the TRE is not compulsory. You can browse and download resources without registering. You will, however, need to register if you would like to submit new resources and add comments or materials to existing resources.

Registration also provides you with a personal profile based on the specific interests selected. For example, a mathematics teacher logging on to the TRE will see a home page drawing attention to all new mathematics resources on the site (over 300 and rising). It is possible to amend the registration profile at any time. Your personal homepage also provides easy access to any resources submitted by you.

The BECTa Educational Software Database (BESD)

BESD contains information about software from pre-school to further education. Software included in the BESD is educational in nature and available in the UK.

The basic search will allow you to search by subject area, phrase, platform, format or keyword. A ‘Help’ facility is available.

BESD information has been provided directly by software publishers. Each of the product records may contain an outline of system requirements, formats, networking options, a description of the software and references to reviews. Where available, there are also links to reviews of the product on the TEEM (Teachers Evaluating Educational Multimedia) Web site. Publisher contact details are included for most records with links to publisher and suppliers web sites.
The ICT Advice Site

The ICT advice web site offers a new concept in ICT support and advice to education professionals working in schools. Aimed at classroom teachers, leadership teams, heads of departments, subject co-ordinators, ICT co-ordinators and special needs co-ordinators, this new site is described as ‘one-stop shop for all ICT education needs’.

As a practitioner you will find a dedicated set of tools to provide access to advice and support that is relevant to your specific professional needs, including classroom and management applications, purchasing, sourcing and planning, technology and inclusion, and teaching and learning.

The site caters for a wide range of experience and skill, so you don’t need to be a computer wizard to take advantage of the advice and support available on the ICT advice site. Your school might, for example, be still in the early days of its ICT development, in which case a visit to the A-Z Guide will provide you with a glossary of concise definitions and terms to help you get to grips with new ideas and applications.

The Timesaver feature is specifically designed to help you find solutions to ICT questions, and has been designed with teacher workloads very much in mind. This contains specific advice to help in evaluating the ICT provision of a mathematics department.

For more detailed descriptions of technology or ICT-related subjects, click on the ‘What is’ button. This will provide answers to a wide range of questions – for example, ‘What is a digital camera?’ or ‘What is a computer virus?’ If you want support to achieve a task which involves using technology, the ‘How to’ guides give clear and concise instructions.

The BECTa Website

The BECTa web site provides information, advice and dialogue relating to ICT in education for the schools and FE sectors. It includes information from case studies and examples of good practice to practical guidance on using ICT within the curriculum and for administration.

Further Education Resources for Learning (FERL)

The FERL web site contains a wealth of information for lecturers, trainers, managers, ILT Champions and support staff; in fact anyone involved in post compulsory education. Despite the ‘FE’ reference in the title there is much here, particularly in the ‘Teaching and Learning’ section, which will be of value to teachers in schools.

There are over 100 resources for supporting mathematics teaching and learning.

The report of a seminar, March 2002, to disseminate the findings of various BECTa projects is at http://ferl.becta.org.uk/display.cfm?page=25&catid=36&resID=2720
The reports and resources, including downloadable software for both Casio and TI technology can be found at http://curriculum.becta.org.uk/docserver.php?docid=4445
Resources to support this application of technology are available from http://curriculum.becta.org.uk/docserver.php?docid=1760
Further ideas including a report from a project on the use of graphical calculators for data capture and modelling in mathematics and science are available at http://curriculum.becta.org.uk/docserver.php?temid=292
The CensusAtSchool Project

The Royal Statistical Society (RSS) Centre for Statistical Education, based at Nottingham Trent University, started the CensusAtSchool project in 2000 in conjunction with National Statistics. In addition it was funded by the Maths Year 2000 initiative and the RSS itself. The project is on-going and, by continuing it, the Centre has three main aims:
1) to provide real data for data-handling activities across the National Curriculum;
2) to increase awareness of what a national census is, and what it is for;
3) to show how Information and Communications Technology (ICT) can be used effectively to enhance learning and teaching resources for good practice in data handling.

Between Autumn 2000 and March 2001, across England, Wales and Northern Ireland thousands of young people between the ages of 7 to 16 took part in the project through their schools, using the Internet site www.censusatschool.ntu.ac.uk. The CensusAtSchool questionnaire itself comprises a single A4 sheet with simple questions covering information about pupils, their households and their school life. While some of the questions were identical to those in the UK adult census that took place in April 2001, others were designed to appeal to the pupil’s own interests and enthusiasms. Over 2000 primary, secondary and special schools registered for the project and there is now a database of over 60,000 responses with excellent coverage of all regions in England, Wales and Northern Ireland. The project can be run equally well using paper methods, computer technology or a combination of the two.

The questions posed can be asked of school children across the globe with minor adjustments to reflect local culture and traditions. For example, in Queensland, Australia the Office for Economic and Statistics Research have carried out the CensusAtSchool project with huge success, and their website carries similar curriculum resources and access to data as the UK one.

The intention is to run a further phase of the project each year so that schools can always become involved in the full data handling cycle and since late Autumn 2001 Phase Two of the project has been running. The phase two questionnaire, which can be completed by pupils on-line using a web form, comprises just 12 questions that link together the international variations of the questions in the original phase 1.

There is a vast selection of resources for educational purposes that can be downloaded from the website at http://www.censusatschool.ntu.ac.uk. These currently comprise:

1) A large, and expanding portfolio of worksheets for Key Stages 2, 3 and 4 which focus on the use of ICT and data handling skills across the school curriculum. Subject areas covered include Mathematics and Statistics, Geography, History, Science, ICT and Key
Skills. These documents can be viewed and freely downloaded after browsing the web site at http://www.censusatschool.ntu.ac.uk/curriculum.asp. All these worksheets are in both Acrobat .pdf and Word .doc formats, the former being visually appealing, while you can choose to alter the latter to suit individual or local needs;

2) More ideas, lesson plans and help sheets in the teachers notes section;

3) Hundreds of tables of summary data that can be easily copied and pasted into Word or Excel for use in the classroom;

4) A variety of Excel Spreadsheets containing sections of data for use in school projects;

5) A random data selector where random samples of raw data can be requested for use in the classroom from four databases from the UK, Queensland, South Africa and Combined Fields. These data are returned as a CSV file attachment so they can be easily imported into commonly used spreadsheets.

6) Links to sister sites in Queensland and South Africa.

Currently work is on the production of interactive web-based curriculum content and a worksheet generator capable of giving you sets of worksheets, each with individual data and answers, that can be created using only a web browser.

The worldwide database, from which random samples may be taken for use in creating learning and teaching materials, is available from the web site and, as more countries participate, it will be expanded accordingly. This will enable the enhancement of teaching and learning data handling in many countries. The involvement of these countries vastly increases the potential for exchange of information between school-aged children and is a unique way to assess global, social and other changes. It has the added bonus of providing ICT, education and motivational resources for both teachers and children.

Additional sources of information

Journals
Mathematics in School (MA), http://www.m-a.org.uk/eb/mis/
Micromath (ATM), http://www.atm.org.uk/journals/micromath.html
Teaching Mathematics and Its Applications (IMA), http://www3.oup.co.uk/teamat/

Calculator support
A+B books (Alan Graham and Barrie Galpin) http://www.fineshade.u-net.com/a+b/

Computer Algebra
Goldstein, R. Computer Algebra at A Level ATM
Oldknow, A. & Flower, J. (1996), Symbolic Manipulation by Computers and Calculators
Terence Etchells web-site (LJMU) http://www.cms.livjm.ac.uk/etchells/

Small programs
Mad maths web-site http://www.madmaths.net/
SMILE Mathematics http://www.smilemathematics.co.uk
Programming languages
MSW Logo web-site  http://www.softtronix.com/logo.html
Programs for TI graphical calculators  http://www.ticalc.org/
**TrueBASIC**  http://www.truebasic.com/demos.html

Spreadsheets

Graph plotters
*Autograph* web-site  http://www.autograph-math.com/
*Coypu* web-site  http://www.octpen.demon.co.uk/coypu/
Omnigraph web-site  http://www.spasoft.co.uk/omnigraph.html
Peanuts software  http://math.exeter.edu/rparris/default.html
**TI Interactive!**  http://education.ti.com/product/software/ii/features/features.html

Dynamic Geometry Software
Geometer’s Sketchpad web-site  http://www.keypress.com/sketchpad/index.html
Michael De Villiers web-site  http://www-cabri.imag.fr/index-e.html

Data-handling
Fathom web-site  http://www.keypress.com/fathom/

Graphical Calculators and Data Loggers
Oldknow, A. (2001) 'The graphic calculator is dead, long live hand held technology’,  *Micromath* 17/3 Autumn 2001
*Secondary Maths with a graphic calculator* (NCET) – in the IT Maths Pack (MA/ATM)
**TI-Time** journal  http://education.ti.com/uk

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 Integrated Learning Systems

General books and reports
BECTa (2002) Using web-based resources in the daily mathematics lesson  
http://www.ictadvice.org.uk
http://www.ofsted.gov.uk
http://www.tctrust.org.uk/resources/publicationsp.html

Suppliers of software and other ICT resources for mathematics
Chartwell-Yorke  
http://www.chartwellyorke.com
114 High Street, Belmont, Bolton BL7 8AL  
info@chartwellyorke.com  (tel) 01204 811001  (fax) 01204 811008

Oxford Educational Supplies  
http://www.oxford-educational.co.uk
Unit 19, Weston Business Park, Weston on the Green, Bicester, OX6 8SY  
dpinfo@oxford-educational.co.uk  (tel): 01869 344 500

QED/Griffin  
http://www.qedgriffin.com
QED Books & Griffin Multimedia, Pentagon Place, 195b Berkhamsted Road, Chesham HP5 3AP  
ged@talk21.com  (tel) 0345 402275  (fax) 01494 793951

Science Studio Limited  
http://www.sciencestudio.co.uk/
Hanborough Park, Long Hanborough, Witney OX29 8SF  
calculators@sciencestudio.co.uk  (tel) 01993 883 598 (fax) 01993 883 317

SMILE Mathematics  
http://www.smilemathematics.co.uk/
Isaac Newton Centre, 108A Lancaster Road, London W11 1QS  
info@smilemathematics.co.uk  (tel) 020 7598 4841  (fax) 020 7598 4838
Finally, a reminder of information contained in Section 1 of the KS3 Strategy’s Framework for Mathematics, page 25.

**Information and communication technology (ICT)**

ICT includes calculators and extends to the whole range of audiovisual aids, including educational broadcasts and video film. The National Curriculum suggests points at which ICT can be used in mathematics. For example:

- Generate functions from plots of data, for example, from a science experiment, using simple curve fitting techniques on graphic calculators, or with graphics software to explore the transformation of graphs.  
  - The National Curriculum 1999

The main uses of ICT in mathematics in Key Stage 3 stem from:
- the use of calculators for calculating purposes (see page 12);
- small programs, such as number games or investigations in a particular context;
- programming languages, such as Logo or Basic, and the programming capabilities of graphical calculators;
- general-purpose software, particularly spreadsheets, but also databases;
- content-free mathematics software, such as graph plotters, dynamic geometry software and data-handling packages;
- ILS (Independent Learning Systems), which provide and manage practice in mathematical techniques tailored to the needs of individual pupils;
- graphical calculators and data-loggers;
- CD-ROMs and the Internet.

The supplement of examples includes some references to ICT. Small programs are not mentioned by name, since the focus in the supplement is on mathematical outcomes, not the resources that can be used to achieve them. Where appropriate, you could add references to the small programs and other ICT resources that you have access to.

**ICT training and support**

The New Opportunities Fund, which will continue to 2003, will provide most teachers with about 30 hours of subject training on the use of ICT, tailored to their individual needs. To help identify training needs, the Teacher Training Agency (TTA) has produced a CD-ROM of four case studies of different mathematics lessons in Years 7 to 10, in which dynamic geometry software, a number game, a spreadsheet and graphical calculators are used. The case studies also show different ways of projecting screen images to whole classes.

The National Grid for Learning (NGfL) aims to connect all schools to the Internet. A major element of the NGfL is the Virtual Teachers’ Centre, which includes useful information, resources and advice for mathematics teachers. There is more useful information, including reviews of software, on the websites of The Mathematical Association at www.m-a.org.uk and The Association of Teachers of Mathematics at www.atm.org.uk.

**Funding for hardware and software**

The Standards Fund can be used to buy hardware and software. It also supports teachers’ use of ICT for their professional development and for administrative work associated with teaching. You can buy:
- ICT equipment, including class sets of portable computing devices such as graphical calculators, whole-class teaching aids such as projection equipment, or equipment to help create teaching materials;
- technical training which complements but does not replicate the pedagogical training supported by the New Opportunities Fund (see above);
- software related to mathematics, including online resources and site licences.

In addition, the pages of Y789 Examples in the Framework contain very many references to the use of graphical calculators, dynamic geometry software, data-loggers and other ICT tools in mathematics throughout the Key Stage.
3.2 Support for teachers

The National Lottery funded (NOF) programme of ICT training for teachers is currently in its last year of operation. Here we give information about (a) an alternative model provided by the Mathematical Association, (b) the government’s plans for a new online CPD service to support ICT in KS3 mathematics and (c) a CPD scheme developed by the Royal Statistical Society specifically concerned with data handling in the curriculum. There are other opportunities provided locally e.g. by LEAs, as well as by commercial companies.

The T-cubed Programme  

T³ (Teachers Teaching with Technology) is the Mathematical Association’s programme of ICT professional development. Ros Hyde is the co-ordinator for England, Wales and N. Ireland by: hyde@tcp.co.uk. Information can be found on the MA web-site. Its coordinating group includes members of ACITT, ASE, ATM, NAMA and NANAMIC. T³ supported the pilot schools on the 2000/1 DfEE Year 7 mathematics/ICT MathsAlive! project, managed by Research Machines. Groups of teachers, consultants, advisers etc. can apply to mount a T³ course or workshop. Provided these meet certain criteria of numbers and duration, T³ provides master copies of course materials free of charge for course organisers to duplicate for participants. It also provides an experienced tutor, whose fee and expenses will be normally met by the T³ programme. Equipment for use by participants in the course/workshop can be provided on loan free from the TI Workshop Loan Programme (see below) - which also covers the delivery and collection charges. Participants in courses/workshops, including the organiser(s), can also purchase related hardware and software products at a considerable discount through the T³ grant pricing scheme. They also qualify as ‘Super-users’ and can access a range of free PC software and GC Applications. For information about mounting a T³ course/workshop consult the MA web-site, or contact Ros Hyde.

The TI Workshop Loan Programme  
[http://education.ti.com/calc/uk/loan.htm](http://education.ti.com/calc/uk/loan.htm)

Loan requests can be made via the web-site, or via ti-loan@ti.com, (tel) 020 8230 3184, (fax) 020 8230 3132. All the equipment for a workshop or course can be borrowed, usually for a period of up to two weeks. Delivery is made two or three days beforehand, and instructions are provided on how to return the equipment by pre-paid courier after the loan period. For additional information about TI’s educational service contact Melanie Horsburgh, Marketing Manager, UK and Ireland, Texas Instruments, 800 Pavilion Drive, Northampton, NN4 7XL Tel: 01604 663059 Fax: 01604 663004 mhorsburgh@ti.com. Other useful contacts are: education.ti.com/uk Customer Service Centre: 020 8230 3184, and e-mail help from ti-cares@ti.com

The T³ Grant Pricing Scheme

Educators who have attended a T³ workshop or course are entitled to buy single copies of relevant equipment at a substantial discount. These include the following:

- TI 83-Plus Silver Edition (2Mb) teacher's VS calculator
- TI-83 ViewScreen for use with OHP
- CBR motion detector data-logger
- TI Presenter video interface for TV/data-projector
- GraphLink cable for USB port
- CBL2 versatile data-logger with 3 probes
- TI Interactive! and other PC software.
Online CPD and support materials in the use of ICT in KS3 mathematics teaching

The DfES have announced a competition to provide this service, following on from a pilot which involved the development of on-line support for KS1/2 numeracy and literacy, as well as KS3 Science. The DfES announcement (July 2002) includes the following:

2.1 ICT occupies an important position as a subject of the National Curriculum. As schools’ provision of ICT improves, it is becoming increasingly important as a teaching and learning tool to enhance pupils’ learning experience and raise levels of attainment in all subjects. Teachers’ ICT capability is of central importance to the effective delivery of the National Curriculum. This was recognised several years ago, and was the reason for the introduction in 1998 of the Initial Teacher Training National Curriculum in the use of ICT in subject teaching. To address the training needs of the force of serving teachers, the New Opportunities Fund is drawing on £230 million of proceeds from the National Lottery to provide all serving teachers in the maintained sector with the opportunity to receive ICT training comparable to the training received by teachers now entering the profession.

2.2 Whilst important, this initial ICT training is only the first stage in the development of teachers’ ICT capability and further steps need to be taken to raise teachers’ levels of competence and confidence in the use of ICT in subject teaching and learning. The development of higher order ICT skills, building on initial teacher and NOF training, must be regarded as a key part of teachers’ continuing professional development. The Department has a responsibility to address these further training and development needs. During the 2001/02 academic year pilots were run for a number of subject areas and the Department for Education and Skills are now looking to ‘roll-out’ the pilots nationally. It is in the context of this national roll-out that the present procurement of online CPD is being carried out.

2.3 This competition covers the development and piloting of materials in English and mathematics at Key Stage 3. The contractor(s) will be expected to consider making available an integrated package of support materials, which must include online web based material, and may include paper-based materials, CD-ROMs, DVDs, or videos. Resources need to be complementary to classroom lessons, and their aim is to support interactive teaching. It is envisaged that the online material will also support the strengthening of teachers’ subject knowledge. The contractor(s) should develop this as an integral part of the package by developing high quality online teacher training material using presentations with text, sound, graphics, video and animations.

2.5 In the case of the projects in English and mathematics at key stage 3 the strategies’ network of consultants will support the development of materials and their rollout to selected schools.

2.6 The use, by teachers, of the materials developed by this project will not be compulsory. It will therefore be essential that the materials are engaging, easily accessible and understood, and wholly relevant to teachers’ individual needs. The proposals should, therefore, include innovative and exciting ways of delivering these materials, and should include a range of strategies to deliver the aims of the project.

2.7 The materials and associated support models should:
- Build on the outcomes of the New Opportunities Fund (NOF) ICT teacher training, whilst recognising that teachers will have varied levels of expertise;
- Deliver training online and meet the popular demand by teachers to have face to face/‘human’ support;
• Draw on the lessons learnt through NOF training for the most preferred and effective methods of teaching;
• Involve the Key Stage 3 English and mathematics consultants in the dissemination and evaluation of the training materials, whilst also developing a model which can be employed in the delivery of training in subjects which do not have such support;
• Link directly to the teaching objectives and classroom approaches of the Key Stage 3 English and mathematics strategies and the strategies of any other subjects covered where relevant;
• Include plans, using existing delivery mechanisms, for teachers to exchange information regarding the use of ICT in their particular subject area, to share good practice and to have the facility to engage in discussions with colleagues online; and
• Make use of existing online resources already available.

2.8 The successful contractor(s) should work closely with the strategies and the subject associations in meeting these aims.

The pilot projects are scheduled to begin work in January 2003.

Using ICT and Web Based Materials for Learning & Teaching and Data Handling Across the Curriculum in Schools

This course developed by the Royal Statistical Society (RSS) Centre for Statistical Education wraps ICT training around data handling specifications across the national curriculum. It uses the international web-based teaching resource the CensusAtSchool project to show how to integrate the use of software, the Internet and web-based resources into the learning and teaching environment. There is significant hands-on experience with computer-based tools such as word processing, spreadsheets and use of the Internet, using them to develop existing or new web-based teaching and learning resources. The course is based upon a two-stage approach: stage 1 is an interactive teacher course and stage 2 involves self-learning and home activities. Teachers use their own specialist curriculum knowledge to develop web-based teaching resources specific to their own disciplines. The length of the course was originally designed to last two days but is very flexible and arranged into a number of sessions that can be adapted to suit most time constraints.

The underpinning for material in this course is data handling, since it is a topic that is common to many core curriculum subjects. As stated in the National Numeracy Strategy, data handling is best taught in the context of real statistical enquiries, in a coherent way so that teaching objectives arise naturally from the whole cycle, as represented in the following diagram:
The new GCSE Mathematics syllabus for first examination in 2003 requires candidates to produce one piece of extended work that demonstrates an ability to handle and compare data. This is worth 10% of the total marks. This data handling project also offers opportunities for candidates to produce a piece of work that will satisfy the vast majority of requirements for Key Skills in both Application of Number and Information Technology (provided ICT methods are utilised) at level 1 or 2 and indeed much of level 3 of Application of Number.

The aims of the course are:

• training teachers in both data handling and ICT skills, linking these together;
• empowering teachers with confidence and transferable skills in these areas;
• using CensusAtSchool as a way to involve pupils fully in the data handling cycle;
• helping the professional and personal development of teachers through interactive training, with special emphasis on web-based resources;
• appreciating the need to collect data using samples or censuses, in order to make informed decisions;
• providing a framework for the creation of information technology-based and traditional learning and teaching material across a broad range of curriculum subjects, and the actual creation of web-based and other technology-based teaching resources for sharing.

For further information contact Doreen Connor at The Royal Statistical Society Centre for Statistical Education, Nottingham Trent University, Nottingham UK
Phone +44 115 848 4471 Fax: +44 115 848 4771 http://science.ntu.ac.uk/rsscse

3.3 Past, and ongoing, trials of ICT and where to find about them

This introduction is a brief summary of some of the range of projects, initiatives and reports on the use of ICT to support the teaching and learning of mathematics from the past decade, where there has been an outcome which is still accessible and relevant.

A resource which is still of interest is the set of video clips of teachers using ICT in mathematics available on the ‘Identification of Needs’ CD ROM from the TTA. This is still available from the TTA publishers. The TTA website is also a source for the reports by teachers who have received research grants for the use of ICT in mathematics.

Other projects which have produced resources which are still available are the:

• GEST funded projects in 1993-1999. Reports and advice from schools involved in these are available on the ICT support pages of the Virtual Teacher Centre (VTC) http://curriculum.becta.org.uk/docserver.php?temid=89 eg:
  ➢ A simple guide to the variety of experiences which ICT can give pupils in mathematics
  ➢ An account of the development of some mathematics departments who received GEST funding and a departmental audit
  ➢ A guide to data capture and modelling in mathematics and science
  ➢ A report on software available for teaching and learning mathematics
  ➢ Resources for teaching advanced mathematics with graphical calculators
• More recent projects (also available through the ICT support pages on the VTC) by the British Educational Communications and Technology Agency (BECTa) have produced
reports and resources for the use of handheld technology (graphical calculators) running small software for KS3 and KS4

resources for the use of four function calculators for whole class activities (primary focused but with some relevance for KS3)

resources for the use of web based resources for whole class activities (primary focused but with some relevance for KS3)

Some additional useful reports:

- **A summary of research** (including the NCET portables project); [http://www.education.leeds.ac.uk/research/groups/cssme/hennessy.pdf](http://www.education.leeds.ac.uk/research/groups/cssme/hennessy.pdf)
- **Effective pedagogy using ICT for Literacy and Numeracy** (in Primary but still relevant) [http://www.ncl.ac.uk/education/taict](http://www.ncl.ac.uk/education/taict)
- **BECTa ILS (Integrated Learning System) evaluation**: from BECTa publications: BECTa, Milburn Hill Rd, Science Park, Coventry CV4 7JJ
- **Examples of effective use of ICT in subjects**: [http://rm.com/_RMVirtual/media/Downloads/good_practice_ICT.pdf](http://rm.com/_RMVirtual/media/Downloads/good_practice_ICT.pdf)
- **Advanced Calculators in Mathematics Education** (Scottish CCC) [http://www.svte.org.uk/scce/acme](http://www.svte.org.uk/scce/acme)
- **Royal Society reports on Algebra and Geometry** [http://www.royal.soc.ac.uk/education/index.htm](http://www.royal.soc.ac.uk/education/index.htm)
- **ImpaCT2 report: Emerging Findings from the Evaluation of the Impact of Information and Communications Technologies on Pupil Attainment** (NGfL) [http://www.becta.org.uk/impact2](http://www.becta.org.uk/impact2)
- **DfEE Towards the classroom of the future – ideas in action**
- **NGfL Transforming the Way we Learn – a vision for the future of ICT in schools** [http://www.dfes.gov.uk/ictfutures](http://www.dfes.gov.uk/ictfutures)

Current developments

- The BECTa/QCA exemplification website of ICT in the mathematics curriculum will be launched soon.
- The Key Stage 3 strategy has asked the MA to produce sample lessons of how objectives from KS3 ICT Framework can be achieved within mathematics lessons taught to the KS3 Mathematics Framework.
- The DfES has announced a competition for pilot projects to provide on-line support for KS3 mathematics in the use of ICT for teaching mathematics.
- The DfES Y7 pilot project has been developed by RM into a full KS3 service called MathsAlive Framework Edition.
- NANAMIC, TI and T-cubed have a project to produce resources to support a variety of post-16 courses using **TI Interactive!** (MEI modules, IB modules, FSMU, AS Use of Mathematics).
- The TCT, TI and T-cubed are establishing a similar project for KS3 and GCSE.
- The DfES has a programme called Transforming Teaching and Learning which is supporting the production of reports, research and examples.
- The 21st Century Citizen project is to be launched at BETT 2003. This will include web based resources for teaching citizenship through statistical activities from the Office of
National Statistics, the Royal Statistical Society, the British Library and the Public Record Office.

- BECTa will be producing the Maths Online publication.
- A variety of new ‘Apps’ for graphical calculators are being developed including versions of Cabri, Logo, SMILE programs etc.

3.4 Future developments

Hand held technology such as graphical calculators, personal digital assistants (PDAs), palm top computers and mobile phones are all evolving rapidly. There is some convergence happening too - for example, all of these will soon be able to communicate wirelessly with other technology, including large scale displays and Internet servers. While most of this (with the exception of GCs) is driven by the needs of business users, there are ways in which this development may well have an impact in the classroom. Some schools already have a wireless network of GCs which can communicate with each other, PCs, large scale displays (interactive white boards) and the Internet. See, for example, the article by Nevil Hopley ‘Calculator networks’ Micromath 18/1, Spring 2002 and information on the TI Navigator system at http://education.ti.com/product/tech/tinav/overview/overview.html.

Some schools and colleges are experimenting with giving teachers and pupils PDAs, mainly as a way of impacting on workload and personal productivity - but there are also teaching and learning developments. For example, some PDAs can be linked to projectors and can be used to run Power Point presentations. There is at least one emulator of a graphical calculator available for a PDA and many of the applications and programs developed for a PC environment will also have versions available for handheld technology too. Some versions of PDAs will act as e-book readers, allowing pupils to carry text books around in electronic versions. Some GCs now have applications which will allow pupils and teachers to use them as personal organisers, storing information such as timetables, appointments and other personal productivity data. Wireless communication also has the potential to impact on teaching and learning as well as on personal productivity.

In a fairly short time this personal technology will be as common as the mobile phone, indeed personal wireless communication will be built into most of these tools. Broad band wireless connections will allow all teachers and pupils to share a vast range of information in their present classrooms, without having to book a computer lab or timetable pupils on a cluster of PCs. Access to ‘curriculum online’ and technology will then be available to teachers and pupils as and when it is needed. Flexible keyboards or voice activated input will provide the means for data to be put into the technology.

There may still be a place for more specialised technology, but displays and facilities are going to be of much higher quality and pupils will be able to use small software programs and other applications without having any special organisation.
3.5 Glossary

AAMT  The Australian Association of Mathematics Teachers
ACITT  The Association for IT in Education – subject association for ICT teachers
AS  Advanced Supplementary – 1 year ‘half A-level’ qualifications
ASE  Association of Science Education
ATM  The Association of Teachers of Mathematics
BASIC  Simple programming language developed for education in the 1960s
BBC  British Broadcasting Corporation – currently developing Curriculum Online
BECTa  British Educational and Communication Technology Agency
BESD  BECTa Educational Software Database
BETT  Educational Technology show held in London each January
BSRLM  British Society for Research in Learning Mathematics
CADCAM  Computer Aided Design and Computer Aided Manufacture
CAS  Computer Algebra System – capable of symbolic manipulation
CBL  Calculator Based Laboratory – multi-purpose data-logger from TI
CBR  Calculator Based Ranger – motion detector from TI
CPD  Continued Professional Development (formerly known as INSET)
CSV  Comma Separated Variables – a form of file structure
D&T  Design and Technology
DES  Department of Education and Science – later became DFEE, then DfES
DFEE  Department for Education and Employment (now DfES)
DfES  Department for Education and Skills
DGS  Dynamic Geometry Software
DISCUS  Discovering Important Statistical Concepts Using Spreadsheets
doc  The file format used by MS Word
DVD  Digital Video Disk
FERL  Further Education Resources for Learning
FSMQ  Free Standing Mathematics Qualifications – post-16 (formerly FSMUnits)
GC  Graphical Calculators
GCSE  General Certificate for Secondary Education
GEST  Grants for Educational Support and Training – replaced by Standards fund
Google  An Internet ‘search-engine’
html  Hypertext Mark-up Language – file format used by MS Internet Explorer
ICT  Information and Communications Technology
IB  The International Baccalaureate – an integrated qualification 16-19
ILS  Integrated Learning System – also Individualised LS or Independent LS
IMA  The Institute of Mathematics and its Applications
INSET  In-service Education of Teachers – now usually called CPD
IT  Information Technology
ITT  Initial Teacher Training
IWB  Interactive White Board
JAVA  A file format used to run applications over the Internet
JMC  The Joint Mathematical Council of the United Kingdom
KS3  Key Stage 3 – i.e. the first 3 years of secondary school = Y7, Y8, Y9
LCD  Liquid Crystal Display
LEA  Local Education Authority
Logo  Simple programming language developed for education in the 1960s
LSP  Learning Schools Programme – a NOF ICT partnership of RM and the OU
LTSN  Learning and Teaching Support Network
MA  The Mathematical Association
MAFE  Maths Alive Framework Edition – a KS3 scheme developed by RM
MEI  Mathematics in Education and Industry
MESU  Microelectronics Support Unit – precursor to NCET
MIT  Massachusetts Institute of Technology
MS  Microsoft
NAMA  National Association of Mathematics Advisers
NANAMIC  National Association for Numeracy and Mathematics in Colleges
NC  National Curriculum
NCET  National Council for Educational Technology – replaced by BECTa
NGfL  National Grid for Learning
NOF  New Opportunities Fund – for distributing national lottery proceeds
NRICH  A mathematics enrichment web-site from Cambridge University
Ofsted  The Office for Standards in Education
OHP  Overhead Projector
OHT  Overhead (projector) Transparency – also called a `foil’
OU  The Open University
OUP  The Oxford University Press
PAL  The standard used in the UK for colour television signals
PC  Personal Computer
PCAI  The Pose, Collect, Analyse & Interpret cycle for data-handling
PDA  Personal Digital Assistant – new breed of hand-held technology
pdf  A file format for documents used by Adobe Acrobat Reader
PE  Physical Education
PHSE  Personal Health and Social Education
QCA  The Qualifications and Curriculum Authority
QED  A supplier of educational software and resources
RM  Research Machines – a supplier of educational ICT
RSS  The Royal Statistical Society
SAT  Standardised Assessment Task – such as those set at KS3
SEN  Special Educational Needs
SMILE  A mathematics curriculum resource and support provider in London
SMP  The School Mathematics Project
SVGA  Super VGA – a standard display signal for PCs, monitors and projectors
T-cubed  Teachers Teaching with Technology – an ICT CPD programme from the MA
TCT  The Technology Colleges Trust
TEEM  Teachers Evaluating Educational Multimedia
TI  Texas Instruments – a supplier of educational ICT
TII  TI Interactive! - integrated mathematics software
TRE  Teacher Resource Exchange
TTA  The Teacher Training Agency
TV  Television
UK  The United Kingdom
VGA  A standard format of display signal for PCs, monitors and projectors
VTC  The Virtual Teachers Centre
XVGA  Extra VGA - a standard display signal for PCs, monitors and projectors